Quantification of the Resolution in the Inversion results from Towed Streamer EM Data

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Summary

The resolution of resistivity models from 3D inversion of Towed Streamer EM data is analyzed using the resolution matrix. In particular, a new resolution length derived from the resolution matrix is introduced. This measure is obtained by dividing the volume of an inversion cell with the corresponding diagonal element in the resolution matrix. Hence, the measure is independent on the actual cell discretization of the inversion domain. Resolution lengths are calculated both for a hypothetical case with noise free data and for resistivity sections obtained with regularized 3D inversion. The resolution of the ideal case with no noise constitutes the upper limit of the best possible resolving power of the Towed Streamer EM data. This limit is then compared against the results where smoothing regularization has been applied in the 3D inversion. A comparison is illustrated for inversion results of Towed Steamer EM data from a survey line in the Barents Sea.

Introduction

Dense spatial data sampling in Controlled Source Electromagnetic (CSEM) acquisition is important in order to achieve high resolution in the resistivity sections from inversion. The effect of various spatial sampling has been studied in Mattsson (2015) by calculating the corresponding resolution and variance matrices for the regularized Gauss-Newton inversion scheme. In the article it is shown that the resolution is dependent on the information contained in the data set such as geometrical location in the model and the model itself etc. It is thoroughly described in Kalscheuer (2010) that first order linearized smoothness-constrained resolution and covariance expressions are representative and accurate estimates of the model variability.

Here we extend the concept of the resolution matrix to the theoretically finest resolvable cell discretization in 3D inversion. This is accomplished by deriving a resolution length from the resolution matrix. It is also demonstrated that an increase in the spatial data density from 1000 m to 500 m between the data points along a survey line improves the data resolution in 3D inversion of Towed Streamer EM data. This effect was shown for 2.5D inversion results in Mattsson (2015).

Quantification of the resolution in 3D inverse modeling

To start with, we will derive the expression for the best possible resolution length in an inversion result. That is inversion without regularization. We then generalize the derivation to include a Gauss-Newton inversion scheme with regularization.

Now, assume an inversion domain shaped as a rectangular grid of N volumetric cells:

$$V_v, \quad v = 1,2,\dots,N \tag{1}$$

The conductivity σ_v is constant within each cell, which can be viewed as an average of $\sigma(x,y,z)$ over each cell V_v. The problem of geophysical data inversion is actually that of solving the inverse operator equation:

$$\mathbf{d} = \mathbf{F}(\mathbf{m}) \tag{2}$$

where

$\mathbf{d}$ = measured data of size M
$\mathbf{m}$ = N logarithmic cell conductivities log (σ_v)
$\mathbf{F}(\mathbf{m})$ = forward modeled data of size M

The true solution $\boldsymbol{m}_{true}$ is assumed to satisfy the equation:

$$\mathbf{d} = \mathbf{F}(\boldsymbol{m}_{true}) + \mathbf{e} \tag{3}$$

where the complex vector **e** includes a noise component of **d,** and discretization errors of the forward model.

The issue of resolution is approached by linear analysis. Linear approximations of (2) and (3) are obtained by Taylor series expansions of $\mathbf{F}(\mathbf{m})$ around a given model vector $\mathbf{m}_k$. If terms of 2nd order smallness in $|\mathbf{m}_{k+1} - \mathbf{m}_k|$ and $|\mathbf{m}_{true} - \mathbf{m}_k|$ are omitted, the linearized systems for (2) and (3) after separation of equations for real and imaginary parts are given by

$$\mathrm{A}\mathbf{m}_{k+1} = \mathbf{b} \tag{4}$$

respectively

$$\mathrm{A}\mathbf{m}_{true} + \mathbf{e} = \mathbf{b} \tag{5}$$

where

$$\mathrm{A} = \begin{bmatrix} ReJ(\mathbf{m}_k) \\ ImJ(\mathbf{m}_k) \end{bmatrix}, \mathbf{b} = \begin{bmatrix} Re\{\mathbf{d} - \mathbf{F}(\mathbf{m}_k) + J(\mathbf{m}_k)\mathbf{m}_k\} \\ Im\{\mathbf{d} - \mathbf{F}(\mathbf{m}_k) + J(\mathbf{m}_k)\mathbf{m}_k\} \end{bmatrix} \quad (6)$$

$$J_{\mu v} = \frac{\partial F_\mu}{\partial m_v}, \quad \mu = 1,2,\dots,M,\ v = 1,2,\dots,N \quad (7)$$

Even if the rank of **A** is 2M and 2M < N there is a unique minimum $l_2 norm$ solution of equation (4) which is given by

$$\mathbf{m}_{k+1} = \mathrm{A}^T(\mathrm{AA}^T)^{-1}\mathbf{b} = \mathrm{A}^\dagger\mathbf{b} \quad (8)$$

Substitution of **b** from equation (5) then gives

$$\mathbf{m}_{k+1} = R_0\mathbf{m}_{true} + \mathrm{A}^\dagger\mathbf{e}, R_0 = \mathrm{A}^\dagger\ \mathrm{A} \quad (9)$$

where R_0 is the resolution matrix of the unregularized problem. The same resolution matrix but for a regularized inverse problem is given in Kalscheuer (2010) and Mattsson (2015) but derived slightly differently.

We will now prove that the cell volumes divided by the corresponding diagonal elements in R_0, i.e. $\frac{|V_v|}{R_{0vv}}$, is a measure of the resolution which is nearly independent of the grid cell size in the inversion domain. The proof starts by making a singular value decomposition of the matrix A:

$$\mathrm{A} = U\Sigma V_I^T \quad (10)$$

where

Σ_{2Mx2M} is a diagonal matrix of singular values.

The singular vectors are columns in the matrices:

$$U_{2Mx2M}, \quad UU^T = U^TU = I_{2Mx2M}$$
$$\mathrm{V}_{I,2MxN}, \quad V_I^TV_I = I_{2Mx2M}$$

From the SVD in (10) it follows that

$$\mathrm{A}^\dagger = V_I\Sigma^{-1}U^T,\ \ R_0 = V_IV_I^T \quad (11)$$

Then, if the noise vector $\boldsymbol{e} = \mathbf{0}$, it follows from (9) and (11) that $\mathbf{m}_{k+1}$ is the orthogonal projection of $\mathbf{m}_{true}$ on the subspace spanned by the columns of V_I. Furthermore, the diagonal elements of R_0 are given by

$$R_{0vv} = \sum_{v'=1}^{2M} V_{Ivv'}^2 \quad (12)$$

which implies that $0 \le R_{0vv} \le 1$ since 2M < N. The case of $R_0 \approx 1$ whould imply that $\mathbf{m}_{k+1} \approx \mathbf{m}_{true}$, where as for $R_{0vv} \ll 1$, $\mathbf{m}_{true}$ is marginally recovered by $\mathbf{m}_{k+1}$.
In order to formulate a length measure, which is independent of the inversion grid cell size, we consider a hypothetical grid with unit cell volume sides, i.e. $\Delta x = \Delta y = \Delta z = 1$m. A singular vector v_c (any column of V_I for a coarse grid and the corresponding vector v_f on the fine hypothetical grid have the same shape apart from a multiplying factor. The reason for this is that both can be viewed as a discretization of a continuous singular vector, which is obtained in the limit of a vanishing cell size. Now the sum of the squares of the vector elements is unity for all singular vectors. On a cell V_v in a grid there are $|V_v|$ unit cells on the fine grid. Therefore

$$v_{cv}^2 \approx |V_v|v_{fv}^2 \quad (13)$$

where v_{fv} denotes an average of v_f over $|V_v|$ unit cells in V_v. The conclusion is that the size of R_{0vv} increases roughly linearly by the cell volume $|V_v|$. Therefore a cell with the volume

$$V_{0v} = \frac{|V_v|}{R_{0vv}} \quad (14)$$

would recover $\mathbf{m}_{true}$ perfectly. It is reasonable to take V_{0v} as a box of the same form as V_v, that is

$$V_{0v} = l_{0vx} \cdot r_{vy}l_{0vx} \cdot r_{vz}l_{0vx},$$
$$r_{vy} = \frac{\Delta y_v}{\Delta x_v},\ \ r_{vz} = \frac{\Delta z_v}{\Delta x_v} \quad (15)$$

which by substitution into (14) results in a resolution length

$$l_{0vx} = \left(\frac{|V_v|}{r_{vy}r_{vz}R_{0vv}}\right)^{1/3}, \quad [m] \quad (16)$$

along the x-coordinate, and resolution lengths

$$l_{0vy} = r_{vy}l_{0vx}, \quad l_{0vz} = r_{vz}l_{0vx}, \quad [\mathrm{m}] \quad (17)$$

in the y- and z-directions, respectively.

With measured data, the resolution is deteriorated by the influence of noise. The ill-conditioned inverse operator in equation (2) and the presence of noise necessitates the use of regularization. Hence, we now consider the regularized inversion formulated as the nonlinear least-squares problem:

$$min_{\mathrm{m}}\left\{\|\mathbf{d} - \mathbf{F}(\mathbf{m})\|^2 + \alpha(\|\delta_x\|^2 + \left\|\delta_y\right\|^2 + \|\delta_z\|^2)\right\}$$

where

$\alpha,\ \alpha > 0$, regularization parameter
$\|\delta_x\|^2 = \sum_{k=1}^{N_z}\sum_{j=1}^{N_y}\sum_{i=1}^{N_x}(m_{ijk} - m_{i-1,jk})^2$
$\left\|\delta_y\right\|^2$ and $\|\delta_z\|^2$ are defined analogously. (18)

Quantification of Resolution in Inversion of Towed Streamer EM Data

The minimization is performed by the Gauss-Newton method. The search for an optimal regularization parameter α is done concurrently at each Gauss-Newton iteration by constructing the so-called L-curve, Kaugmann (1997), and applying a criteria based on the quasi-solution principle. With the regularization present, equation (9) is extended to:

$$\mathbf{m}_{k+1} = R_{\alpha}\mathbf{m}_{true} + \mathrm{A}_{\boldsymbol{\alpha}}{}^{\dagger}\mathbf{e} \qquad (19)$$

where

$$R_{\alpha} = \mathrm{A}_{\boldsymbol{\alpha}}^{\dagger}\mathrm{A} = (\mathrm{A}\mathrm{A}^{\boldsymbol{T}} + \alpha\delta^{T}\delta)^{-\mathbf{1}}\mathrm{A}^{\boldsymbol{T}}\mathrm{A}$$

is the resolution matrix. This is the expression for the resolution matrix derived in Kalscheuer (2010) and Mattsson (2015). The resolution lengths are still defined by the formulas (15) and (16).

Quantification of the resolution on 3D inverted Towed Streamer EM data from the Barents Sea

A resistivity cross section is estimated by 3D inversion of Towed Streamer EM data acquired along a survey line in the Barents Sea. The corresponding resolution matrices and resolution lengths are computed from the inversion results with spatial shot point separations of 500 and 1000 m, respectively.

During the survey, the Towed Streamer EM system consisted of an 800 m long electric bi-pole source towed at a depth of 10 m. A 90 s long Optimized Repeated Sequence (ORS), Mattsson (2012), followed by 30 s silent period was used as the source sequence during a shot. In this case the frequencies used in the inversion were six and ranged from 0.2 to 1.8 Hz. The ORS sequence was repeated for every shot along the survey line. With a towing speed of 4 kn, the resulting distance between each shot was 250m. The resulting electric field was measured in an EM streamer cable towed from the same vessel at a depth of 100 m. The offsets ranged from 1000 to 7500 m. The electric field was then deconvolved with the source output current to obtain the frequency responses of the earth, which are used as the input data to the inversion.

3D inversion of the Towed Streamer EM data

The Towed Streamer EM data were inverted in 3D by minimizing of the functional in (18) using a Gauss-Newton algorithm and an Integral Equation (IE) based forward modeling methodology to solve equation (2) for a given model $\mathbf{m}$. The resulting resistivity cross section along the chosen survey line is shown in figure 1.

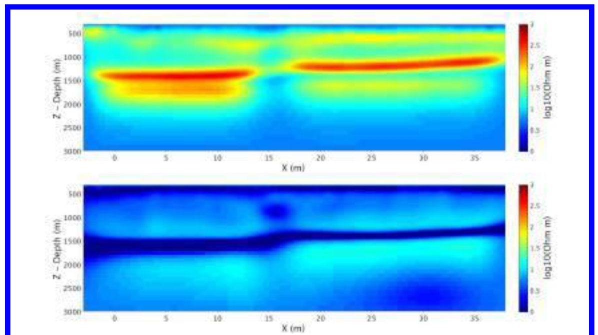

Figure 1: The vertical (top) and horizontal (bottom) resistivity cross-sections resulting from 3D inversion of Towed Streamer EM data with a shot separation of 250 m.

The horizontal and vertical axes in figure 1 show the distance in km along the line and sub-surface depth, respectively. The resistivity values are plotted in a logarithmic color scale.

Resolution of the inversion result

The quality of the inversion result is evaluated by calculating the resolution matrix and resolution lengths as derived above. The final iteration of the inversion is assumed to be close to the true model and thus the linearization leading to equations (9) and (19) is regarded as a good approximation of the otherwise non-linear problem.

The resolution matrix is calculated for two different configurations. First with a shot point separation of 500 m and then for a shot point separation of 1000 m. The resolution matrices are calculated with and without regularization for both configurations.

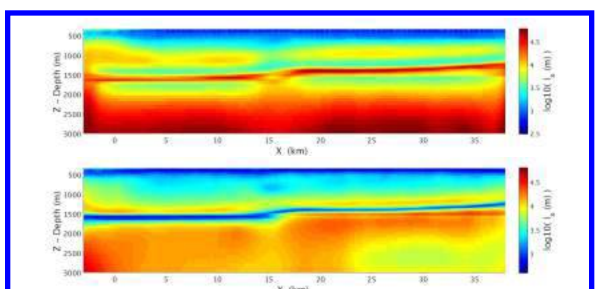

Figure 2: The vertical (top) and horizontal (bottom) resistivity length $\boldsymbol{l_x}$, with a shot separation of 500 m without including the regularization term. The smallest resolvable lengths in the domain are around 400m horizontally and 40m vertically.

Figure 2 shows the resolution length $\boldsymbol{l_x}$ for each volume element in the inversion grid for the vertical and horizontal resistivities from the 500m shot point separation configuration and without regularization present. The highest achievable horizontal resolution length in this configuration is around 400m horizontaly and 40m vertically and is rapidly increasing to many kilometers in the least sensitive areas of the inversion domain.

In figure 3, the resolution for the 500 m shot separation configuration is shown but now with the regularization term included. The resolution length is now roughly increased by a factor of 1.125.

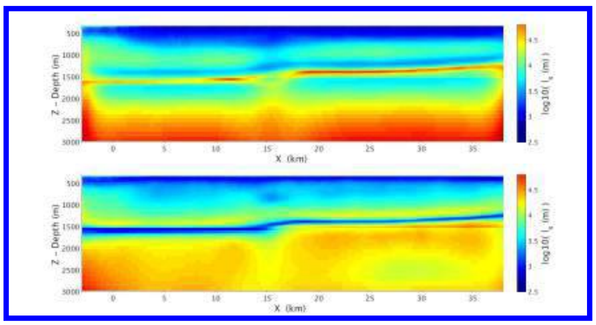

Figure 3: The vertical (top) and horizontal (bottom) resistivity length $\boldsymbol{l_x}$ with a shot separation of 500 m and with the regularization included. The smallest resolvable lengths in the domain are around 450m horizontally and 45m vertically.

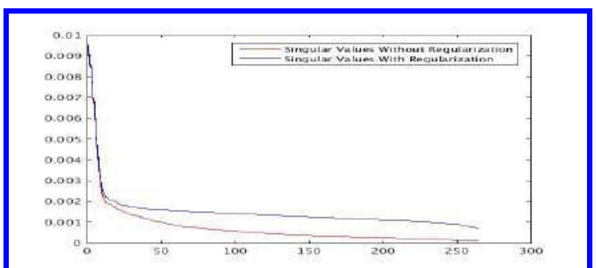

Figure 4: The first 270 singular values of the SVD decomposition of the matrix A with and without regularization.

Typically, the solution to the model update $\mathbf{m}_{k+1}$ is very unstable and some form of regularization must be included in order to find a stable solution. The regularization will increase the number of significant singular values in the SVD of the matrix A. See figure 4. This increase with the regularization present, will decrease the resolution of the inverse problem. This is seen when comparing figures 2 and 3.

The corresponding resolution lengths for the reduced dataset with 1000 m shot separation are shown without and with regularization in figures 5 and 6, respectively. The reduced dataset result in a slightly lower resolution compared to the 500 shot separation configuration. The resolution length in the upper part of the sub-surface has not changed much while the general trend in the deeper part of the domain is showing a slightly lower resolution with increased shot separation. At 1000m depth the factor is about 10% lower resolution for the reduced data.

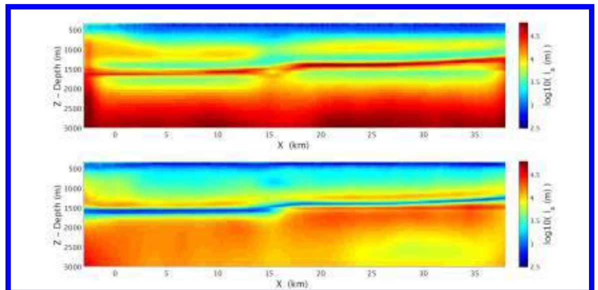

Figure 5: The vertical (top) and horizontal (bottom) resistivity length $\boldsymbol{l_x}$, with a shot separation of 1000 m without including the regularization term. The smallest resolvable length has not changed.

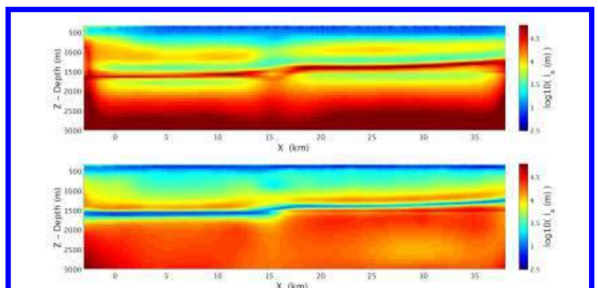

Figure 6: The vertical (top) and horizontal (bottom) resistivity length $\boldsymbol{l_x}$ with a shot separation of 1000 m and with the regularization included. The smallest resolvable length has not changed.

Conclusions

It has been shown that the resolution matrix can be converted to a resolution length which is independent of the inversion grid cell size. Inversion results with two different shot point separations show that the resolution length is increased when the shot separation is doubled, the effect being larger for the regularized problem. The effect of the regularization with respect to the resolution length is also investigated. From the inversion results it is seen that the smallest resolution length is increased with a factor of about 1.125 when including the regularization. However, this factor depends on the weights on the regularization term.

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EDITED REFERENCES

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