Adaptive multiple subtraction by statistical curvelet matching

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Summary

In this paper, we present a novel algorithm that performs adaptive subtraction of a multiple model in the curvelet domain. The algorithm is based on the observation that the predicted model is an imperfect estimate of the actual multiples, containing two types of error: 1. a systematic error that manifests as an approximately constant phase and amplitude error within each subband and 2. a localized error that potentially varies from coefficient to coefficient. Adaptation of the model is automatically controlled by parameters provided by a statistical modelling step. Results show that the algorithm works well with different types of multiples and levels of noise.

Introduction

Multiple attenuation is an important part of a standard seismic processing sequence, usually consisting of two key steps; multiple prediction and adaptive subtraction. The prediction step generates a model of the multiples that is not perfectly accurate, often containing timing, amplitude and wavelet errors. The adaptive subtraction part adjusts the multiple model to improve its accuracy and then subtracts it from the data.

The most widely used adaptive subtraction method is least squares filtering (LSF), which operates in the time/space (TX) domain and adapts the multiple model by convolving it with a filter. This filter is designed to make events in the model optimally similar to events in the data (in an L2-norm sense). This is an effective method, but makes the assumption that multiple and primary events in the data are orthogonal, which is typically not the case when primaries and multiples overlap one another.

Given this drawback of the LSF method, other approaches to adaptive subtraction have been explored. Curvelet transform-based methods have received considerable attention in recent years because curvelets have a number of useful properties that can be leveraged for adaptive subtraction: 1. curvelets provide a sparse representation of seismic events, 2. events of differing dip, scale or TX location will often be well separated in the curvelet domain.

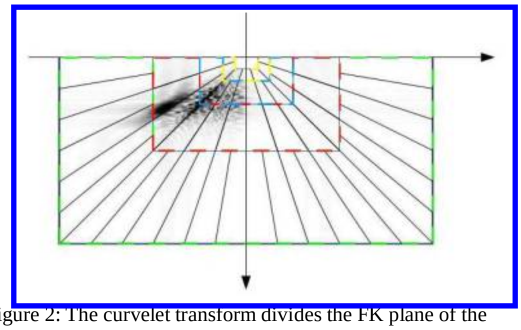

Figure 2: The curvelet transform divides the FK plane of the seismic data in Figure 1 into several scales and directions.

The curvelet transform represents any 2D image as a collection of coefficients (the curvelet domain); this image is expressed exactly as a linear sum of curvelet dictionary functions weighted by corresponding coefficient values. Curvelets are designed to be simultaneously localized in scale, direction, space and time. Each curvelet coefficient is thus identified by a multi-index. Figure 1 illustrates that the curvelet coefficients are divided into several scales and that the coefficients within each scale are further partitioned into several directions. Each scale/direction pair is called a subband and consists of a 2D array of complex numbers. Each subband corresponds to a wedge-shaped region of the frequency/wavenumber (FK) space as shown in Figure 2. The location of a coefficient in the 2D subband array corresponds to the relative position of the curvelet dictionary function in TX.

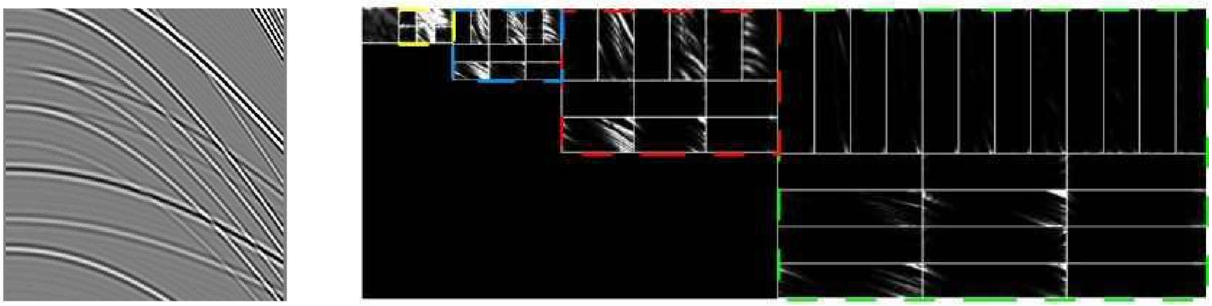

Figure 1: An example of data in the TX domain (left) and the curvelet domain (right). Nguyen *et al.*, (2010)

Curvelet-based adaptive subtraction methods roughly fall under one of two types.

In the first, an explicit objective function is defined with curvelet-domain primaries and multiples as unknowns to solve for. Terms in this objective function are designed to promote sparsity of the unknowns. Additional terms impose agreement with the multiple model and data (Saab *et al.*, 2007). The difficulty with this method is that it is not easy to find a solution to the optimization problem, or to account for the difference between the predicted model and true multiples.

In the second, the multiple model and data are both transformed to the curvelet domain. The coefficients of the multiple model are adjusted to become “closer” to the coefficients of the data. In the FK domain, each complex coefficient corresponds to a plane wave in the TX domain. Varying the phase of the coefficient is equivalent to shifting the wave in the TX domain. Similarly, in the curvelet domain, a small change in the phase of a curvelet coefficient provides an approximate shift in the TX domain. This observation is used in Neelamani *et al.*, (2010), where each complex curvelet coefficient of the predicted model decomposition is rotated and scaled within a limited user-specified range to match to the corresponding coefficient of the data.

Method

In order to estimate the phase and amplitude difference between curvelet coefficients of the multiple model and the true multiples, we experiment on synthetic data, where true and predicted multiples are readily available. Analysis on the histogram of phase differences and amplitude ratios between curvelet coefficients of the true and predicted multiples shows that within each subband the difference can be modelled as a unimodal distribution concentrated around a mean value. The distributions for curvelet subbands with the same size tend to have the same mean and variance. This is because curvelet subbands of the same size correspond to curvelet functions of the same scale and similar direction (mostly horizontal or vertical directions). The difference in shape of the curvelet in predicted and true multiples can be approximated by the same phase shift of curvelet coefficients. It is also observed that the kinematic error in modelling the multiples will increase the variances in phase difference in the curvelet domain.

Using the above observations, we have devised an algorithm that adapts the curvelet coefficients of the multiple model in two passes: global adaptation followed by local adaptation. Each pass is supplemented with parameters derived from a statistical analysis process. A flow diagram of the method is shown in Figure 3.

At the start, the data, **D**, and model, **M**, are both forward curvelet transformed. From there on, the algorithm loops over subbands of coefficients, processing each independently (This could be generalized to groups of subbands.) Let $\mathbf{c_m}$ be a subband of coefficients from the transformed model and $\mathbf{c_d}$ the corresponding subband from the transformed data.

Statistical analysis

Before the statistical analysis can be done, a sample of significant coefficients from $\mathbf{c_m}$ are selected such that the corresponding coefficients in $\mathbf{c_d}$ are mostly primary-free. This selection process is detailed below. It uses the observation that the largest amplitude coefficients in $\mathbf{c_m}$ will be more highly correlated with multiple energy in $\mathbf{c_d}$ than with primary energy.

Having found the sample set, we calculate $(\mathbf{c_d})_i$ / $(\mathbf{c_m})_i$ where i is an index over coefficients in the sample set. In other words, we calculate the amplitude ratios and phase differences. We compute Γ to be the mean amplitude ratio, Φ the mean phase difference, γ the standard deviation of amplitude ratios and ϕ the standard deviation of phase differences. These are provided as control parameters to the global and local adaptation steps.

Selection of coefficients for statistical analysis

We require for the statistical analysis a sample of coefficients that represent only the predicted and actual multiple energy (i.e., largely free from primary overlap). For the model, we can simply select the set of most significant coefficients (in magnitude). However, the set of corresponding coefficients in the data may not contain pure multiple energy. This is especially true if the multiples are very weak (in the case of internal multiples), or the data

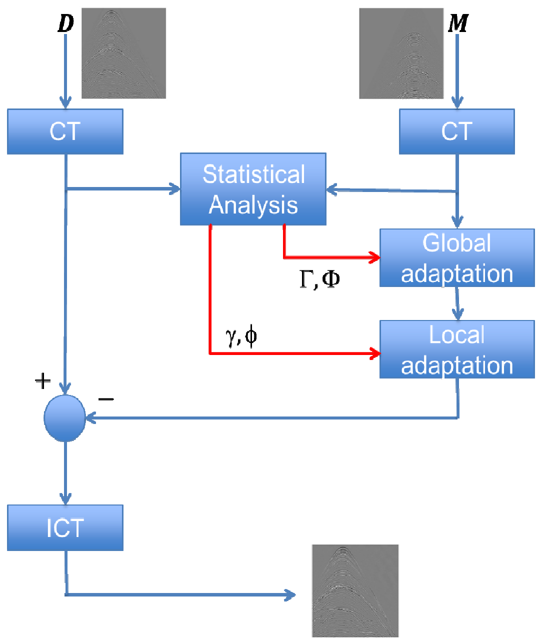

Figure 3 : Data flow of the algorithm

contains strong primaries in the same location. Therefore we use a simple selection algorithm to filter out outliers:

1. For each curvelet subband of the model, select a set of most significant coefficients
2. Estimate the amplitude ratios with corresponding coefficients in the data
3. Classify the ratios into different bins, each bin corresponding to a range of values (histogram)
4. Output the pairs of coefficients belonging to the most populous bin

Global adaptation

The purpose of the global adaptation is to correct for errors in the model that are approximately constant throughout the subband of coefficients. These will correspond to systematic errors in the model such as small timing, amplitude and wavelet errors. The statistical analysis has done most of the hard work, such that the global adaptation simply multiplies each coefficient in $\mathbf{c_m}$ by $\Gamma e^{i\Phi}$.

Local adaptation

The local adaptation step aims to correct for errors that (potentially) vary from coefficient to coefficient. By removing systematic errors during the global adaptation, we can use tighter constraints during the local adaptation, thus reducing the risk of over adaptation and primary loss.

The statistical analysis provides automatic constraints, which would otherwise have to be provided as user parameters. Each coefficient in the model is allowed to phase rotate and scale to optimally match the corresponding coefficient in the data. The amplitude scaling and phase rotation are limited by constraints proportional to γ and ϕ respectively.

Following are the main steps of the algorithm, as illustrated in Figure 3

1. Data **D** and multiple model **M** will be transformed to curvelet domain, $\mathbf{c_d}$ and $\mathbf{c_m}$
2. A number of most significant coefficients of $\mathbf{c_d}$, and the corresponding coefficients in $\mathbf{c_m}$ are selected according to the selection algorithm.
3. For each scale and angle of the curvelet transform used, a statistic model is estimated based on the difference between phase and amplitude of each pair of coefficient belongs to that scale and angle.
4. All the coefficients of each scale and angle of $\mathbf{c_m}$ is transformed to match corresponding coefficients in $\mathbf{c_d}$ by information from the model estimated in step 3. This step consists of global and local adaptation of the model coefficients as described above.
5. The adjusted model coefficients are then subtracted from $\mathbf{c_d}$, the result are in then apply to an inverse curvelet transform to create subtracted data

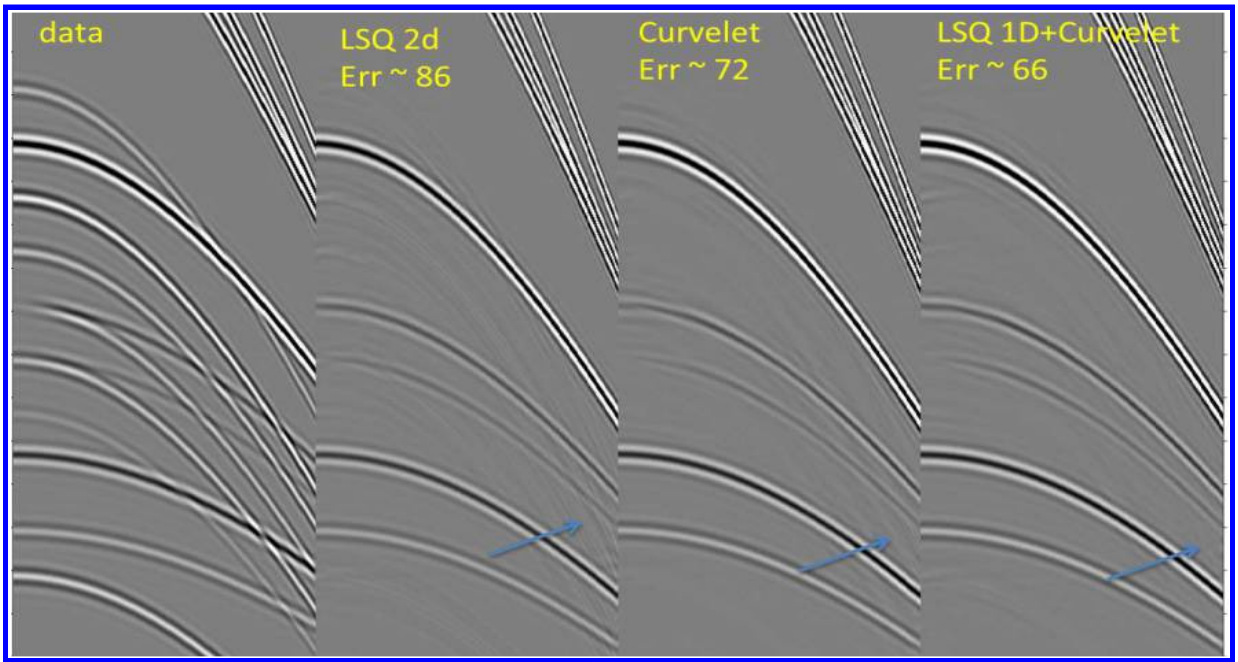

Figure 4 : Example of data with multiple, multiple subtraction by LSF and by the proposed curvelet matching method.

Data examples

We applied our method to several synthetic datasets, varying from simple to complex, comparing against the LSF method and looking for an optimal parameterization and workflow.

In Figure 4, results from curvelet subtraction are compared with results from the LSF method using 2D filters. The predicted multiple model is fairly good, but the 2D LSF algorithm struggles to adapt the model to the actual multiples in a complex area (the highlighted region). Curvelet subtraction has the advantage and can adapt the model to match to real multiples in that area. However, the best result, with minimum error, is achieved when the model is first adapted using a short 1D LSF filter, and then subtracted by curvelet matching. The 1D filter makes events in the model properly aligned with real multiples in terms of location and amplitude. As a result the curvelet subtraction is able to remove the multiples from the data more effectively. This suggest a two-stage workflow, where the LSF module is used as a preconditioning step, whilst the actual subtraction is done by the curvelet subtraction method. In our experiment, this two-stage workflow produces the best result.

In a second experiment, our method was used for adaptive subtraction on the synthetic Pluto dataset from SMAART consortium. With careful parameter selection our workflow produces slightly better results compared to the traditional LSF method. Figure 5 shows the data before multiple removal, and after the use of the LSF, and the two-stage LSF and curvelet subtraction method. The highlighted area is a part of the data which the LSF-based method has traditionally struggled with, because primary and multiple events are nearly parallel. The LSF method tends to over-adapt the predicted multiple wavelets, leading to the attenuation of primary energy. The curvelet subtraction method successfully removes the multiple without damaging the primary event.

Discussion and conclusion

In this work we present a statistical adaptive subtraction method using the curvelet transform. Only significant curvelet coefficients in the model with corresponding data coefficients that are likely to represent only multiples are used in the estimation of a statistical model for each subband. The mean and variance estimated from the models are then used to control how the curvelet coefficients of the model are adapted and subtracted from the data. The mean value is used in a global adaptation step, where coefficients in each subband of the model are multiplied with a complex number. This step is similar to LSF filtering in the curvelet domain (Ventosa *et al.*, 2010). The variance value is used as a constraint in the local adaptation step, where each curvelet coefficient of the model is freely rotated and scaled to match to the corresponding coefficient of the data. This step is similar to the adaptive subtraction step in Neelamani *et al.*, (2010). However, in this original method the range constraint is specified by the user.

Experiments show that our approach can offer improvement in areas where the LSF subtraction method has difficulties. Our method is versatile enough to deal with different types and levels of multiple noises in the data. Curvelet-based multiple subtraction's advantage over traditional LSF subtraction is due to its flexibility in changing the wavelet shape and amplitude of the model to that of the actual multiples, without overspreading the wavelet too much which may lead to over-adaptation.. However it requires a fairly kinematically-accurate prediction of the multiples. A practical workflow is to use the LSF as a preconditioning step before curvelet subtraction. In this way we can maximize the advantages of both methods and have greater flexibility in dealing with the quality of data and model.

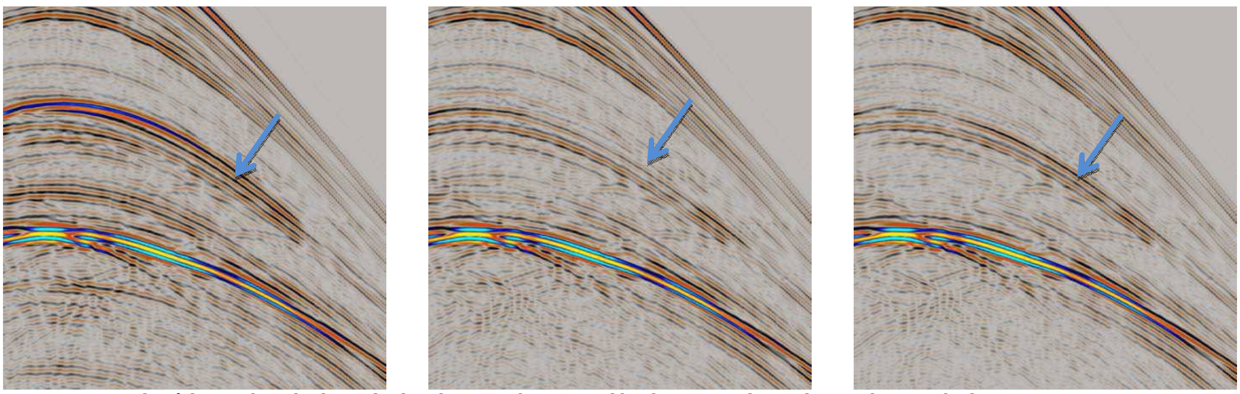

Figure 5: Example of data with multiple, multiple subtraction by LSF and by the proposed curvelet matching method.

EDITED REFERENCES

Note: This reference list is a copyedited version of the reference list submitted by the author. Reference lists for the 2016 SEG Technical Program Expanded Abstracts have been copyedited so that references provided with the online metadata for each paper will achieve a high degree of linking to cited sources that appear on the Web.

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