Full waveform inversion with steerable variation regularization

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Summary

We propose a new regularization scheme for Full Waveform Inversion (FWI). The new method makes use of a priori information on the spatial variability of the earth model to overcome the limitations of the inversion in the presence of high velocity contrast geobodies and cycle skipping. It comprises two additional regularization terms to the FWI objective function. The first term evaluates the $\boldsymbol{L^1}$ norm of total variation (TV) of the model, while the second term steers the solution based on local prior information of the model spatial variability. Both regularization terms can be made spatially variant to accommodate different geological features in the model, i.e. sediments (smooth changes), salt bodies (piecewise constant). Our procedure makes use of the split Bregman iterations, an effective algorithm for solving the $\boldsymbol{L^1}$ optimization problems. The result is a computationally efficient and accurate implementation. We show the potential of the method by using the BP 2004 velocity benchmark model. There, our regularization scheme allows the inversion to start from a simple velocity model and delivers a high-quality reconstruction of salt bodies.

Introduction

Full Waveform Inversion (FWI) (Tarantola, 1984) solves a nonlinear inverse problem by matching modeled data to recorded field data. The matching is quantified by the residuals of a least-squares objective function, and the model update is computed as a scaled representation of its gradient. FWI can produce high-resolution models of the subsurface when compared to ray-based methods. However, FWI is an ill-posed problem due to the band-limited nature of the seismic data and the limitations of the acquisition geometries. Therefore, a regularization procedure is required to steer the FWI solution toward one that is geologically plausible.

One especially challenging case for FWI is a model with high contrast geobodies (e.g. salt, basalt). There the FWI solution gets trapped in local minima if started from a poor velocity model. In this situation, a combination of Total Variation (TV) regularization (Guo and de Hoop, 2013) and the use of vertical hinge-loss asymmetric TV (Esser et al., 2015) can recover relatively complex velocity models from a simple starting model.

Here we discuss an alternative regularization that allows steering the FWI solution in any arbitrary direction based on prior geological information. It combines the variable weighted $\boldsymbol{L^1}$ norm of the total variation (TV) of the model with a weighted version of the model spatial variability. Our approach allows for a generalization of Esser et al. (2015) ideas, as it is not limited to constraining the derivative of the model in the depth direction. The variable regularization parameters allow the refinement of the sediments region of the model with a mild regularization, while promoting sharp contrast and constant velocity geobodies in a different region with a strong regularization. The algorithm is implemented by using the split Bregman method (Goldstein and Osher, 2009). Synthetic results on the BP 2004 benchmark model show that our method recovers the true velocity model where non-regularized FWI fails.

FWI with TV regularization using the split Bregman algorithm

It is well known that the L^1-TV regularization better performs than the L^2 regularization if the model can be well approximated by using piecewise constant functions. The main advantage of the L^1-TV regularization is that the sharp edges are well preserved while the artifacts and noise are efficiently removed during inversion. In other words, the L^1-TV regularization pursues a sparse representation of the model in the space spanned by piecewise constant functions.

FWI with L^1 norm TV regularization can be formulated as the optimization problem

$$\min_m \| F(m) - u\|_2^2 + \lambda \| \nabla m\|_1. \quad (1)$$

Where F is the modeling operator; i.e. the composition of the projection to the receivers and the wave propagation. m is the velocity model, u is the recorded data, and λ is the regularization parameter. Ramos-Martinez et al. (2011) presented a detailed description of our adjoint-state FWI implementation. In this paper we focus on the regularization of the inversion.

The second term in Equation 1 uses the $\boldsymbol{L^1}$ norm to pursue a sparse representation of the high contrast boundaries of the model. The $\boldsymbol{L^1}$ norm can be calculated by using different approximations (e.g. Guitton and Symes, 2003). However, slow convergence has been observed when using those approximations in order to achieve a sparse solution.

Our $\boldsymbol{L^1}$ norm implementation solves the slow convergence problem by using the split Bregman iterations. This method has been proven to be efficient for solving $\boldsymbol{L^1}$ optimization problems, in particular for TV regularization (Goldstein and Osher, 2009). Goldstein and Osher (2009) showed that the optimization problem (Equation 1) is equivalent to

$$\min_m \| F(m)-u\|_2^2 + \lambda \| d\|_1, \text{ such that } d = \nabla m. \quad (2)$$

To weakly enforce the constraints, Equation 2 can be reformulated as

$$\min_{m,d} \| F(m)-u\|_2^2 + \lambda \| d\|_1 + \frac{\gamma}{2}\| d-\nabla m\|_2^2. \quad (3)$$

where we expand the model space with the new variable d. Finally, to enforce the constraint, the Bregman iteration can be applied yielding

$$\min_{m,d} \| F(m)-u\|_2^2 + \lambda \| d\|_1 + \frac{\gamma}{2}\| d-\nabla m - b\|_2^2, \quad (4)$$

where the auxiliary variable b is updated according to

$$b_{k+1} = b_k + \nabla m - d_{k+1}. \quad (5)$$

In a later section we adapt the split Bregman algorithm to accommodate the objective function with the steerable variation term.

Steerable Variation Regularization

The L^1–TV norm regularization with constant regularization parameter (**λ**) treats all regions in the model with homogeneous isotropic weights. Ideally, by including additional constraints, we would like to add any prior physical information about the model to steer the solution in any direction. We call this novel method steerable variation regularization.

FWI with steerable variation regularization can be formulated as the following optimization problem

$$\min_m \| F(m)-u\|_2^2 + \| \lambda \nabla m\|_1 + \int_\Omega \nabla m \cdot P. \quad (6)$$

where the steering field P is used to accommodate the a priori knowledge of the velocity model. The dot product of the gradient of the model and the direction indicated by P can be considered the changing rate of the model along the steering field. Without taking the absolute value, we can not only control the magnitude of ∇m (with the TV), but also guide its direction with the second regularization term. With a careful choice of λ and P, we make sure that the sum of the regularization terms is non-negative.

The field P in the steerable regularization term plays a crucial role in our inversion algorithm. However, it can be refined iteratively as the inversion stage progresses. A source of independent information could be a legacy model or an image from a sediment flood. The latest is one of the earliest deliveries from the velocity model building workflow, so it could be easier to incorporate into our regularization strategy in the absence of a legacy model. To design the FWI steerable regularization workflow, we follow the principle that the goal is to reconstruct the high contrast components in the first stages, and the details can be reconstructed at later stages.

Spatially Variant Regularization Parameters

The regularization parameter (λ) on the TV term (Equation 1) controls the smoothing of the inversion result and has been used in the literature as a constant value (Esser et al., 2015). In practical situations using a constant regularization parameter to accommodate all regions of the model can be suboptimal. Many stages, where the regularization is relaxed, are necessary to define the high contrast events while preserving the high resolution in the sediments. Thus, it is preferable to choose a spatially variant regularization parameter λ for the TV regularization. It provides the flexibility to target the area in which, based on the a priori information, salt bodies may exist without tremendously increasing the computational cost of the inversion.

With a spatially variant regularization parameter, we can build the salt and refine the sediments at the same time without precise knowledge of the salt boundaries position. In that case the salt does not necessarily need to be included in the initial model and rather can be used as a soft constraint in the regularization. The main reason behind this is that the synthetic data is sensitive with respect to the model, but the optimal regularization parameter is not. Hence, if we have inaccurate information on the salt boundaries, it might be preferable to include that prior information in the regularization parameter than in the misfit term.

Implementation of the Steerable Variation Regularization

Following the spirit of the split Bregman algorithm, we can transfer the optimization with steerable variation regularization into an unconstrained optimization problem as in

$$\min_{m,d} \| F(m)-u\|_2^2 + \| \lambda d\|_1 + \int_\Omega d \cdot P. \text{ such that } d = \nabla m. \quad (7)$$

The split Bregman algorithm can be applied to Equation 7 similarly to the plain TV regularization (Equation 2). For the sake of simplicity, we skip the lengthy derivation in this abstract. The final formula is to iteratively solve

$$d_{k+1} = \arg\min \| \lambda d\|_1 + \int_\Omega d\cdot P + \| \frac{\gamma}{2}(d-\nabla m_k - b_k)\|_2^2 \quad (8)$$

$$m_{k+1} = \arg\min \| F(m)-u\|_2^2 + \| \frac{\gamma}{2}(d_{k+1}-\nabla m - b_k)\|_2^2 \quad (9)$$

$$b_{k+1} = b_k + \nabla m_{k+1} - d_{k+1}. \quad (10)$$

This algorithm can also be understood as an alternating direction method of multipliers (P. L. Lions and B. Mercier, 1979). Note that, for the sub-problem to update the velocity model (Equation 9), it actually solves the conventional FWI with the classical L^2 Tikhonov regularization term. Hence, we can make use of the standard FWI procedure (Ramos-Martinez et al., 2011) with minimal modifications.

Numerical Experiment

We investigated the use of steerable variation regularization by using a modified version of the BP 2004 benchmark velocity model (Figure 1a) (Billette and Brandsberg-Dahl, 2005). The model contains two salt bodies with unique characteristics (different velocity values, size, and geometry). These properties promote better illumination, by the refracted/diving waves, of the "tooth" salt body to the right of the model than for the salt body to the left. The synthetic data was created with a minimum frequency of 3 Hz and a maximum offset of 12 km.

The starting velocity model for the FWI consists of a modified version of the low wavenumber model from the benchmark distribution, continued from the center to the right as a V(z) (Figure 1b). Note that the initial model does not contain any high contrast velocity information. At the same time it is not that far away from the sediments true velocity, with the exception of the right of the model which is V(z).

Several stages were used to improve the convergence of the FWI and avoid local minima by starting at low frequencies and working up to higher frequencies. Center frequencies of 5 Hz, 9 Hz, 12 Hz, 15 Hz and 18 Hz were utilized in our time domain code. The same number of frequency bands and number of iterations were employed for all the compared algorithms. For the FWI with TV regularization, a constant regularization parameter was applied with the same decay rate as in the steerable variation regularization scheme. Using a stronger regularization in the initial stages and a milder regularization towards the end.

Figures 1c, 1d, and 1e show the inversion results with and without regularization. Figure 1c displays the result of FWI without regularization, Figure 1d shows the result of the FWI with TV regularization and constant regularization parameter, and Figure 1e shows our steerable variance regularization result. Note how significant artifacts due to cycle skipping are observed in the result using conventional FWI. With TV regularization and a proper choice of the regularization parameter, the artifacts can be reduced and the result is improved but the cycle skipping is still visible. Our FWI with steerable regularization obtains the best result by defining the top and bottom salt boundaries and the correct velocity of the sediments bellow salt. In the right side of the model the conclusions from comparing the different regularizations are the same as the left side of the model, even though the starting velocity is farther away from the true model.

Conclusions and Discussion

We have shown a new FWI regularization scheme that overcomes the limitations of the inversion in the presence of high contrast geobodies and cycle skipping. It allows the use of prior information about the earth model in the regularization as an extra term in the objective function. The implementation makes use of the split Bregman method making it efficient and accurate.

The numerical experiments demonstrate that our algorithm can deal with the challenges of the presence of high contrast geobodies and cycle skipping. We show how the regularization terms can drive the solution out of local minima. It remains to be demonstrated that this kind of constraint on the solution can help the inversion deal with the same problems when using field data. The errors in the physics used for the modeling operator and the noise in the data could demand the use of a better approximation of the wave propagation in the subsurface as well as better data selection techniques.

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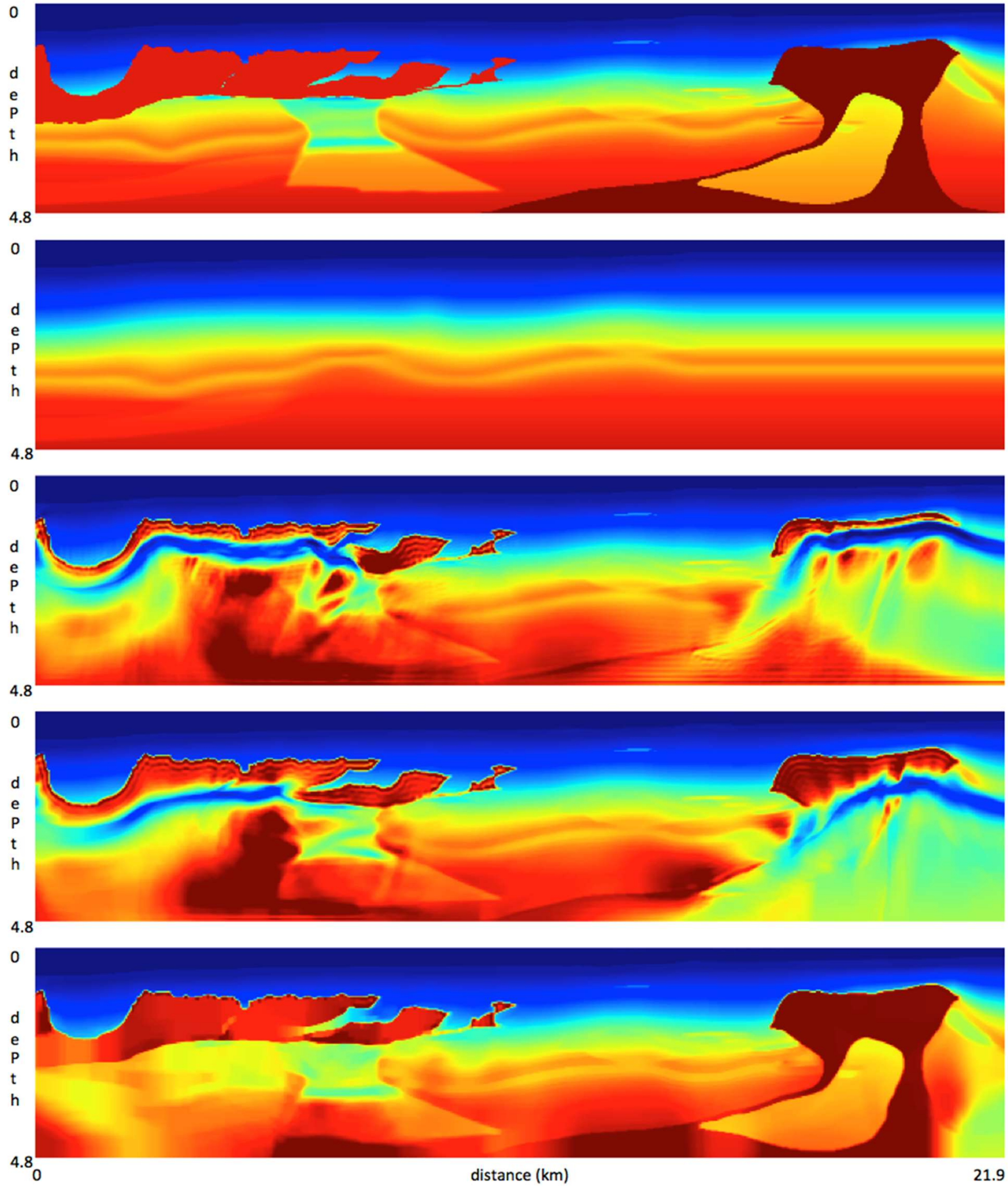

Figure 1: Comparison of different regularization methods. (a) True model, (b) Starting model, (c) FWI without regularization, (d) FWI with TV regularization and (e) FWI with steerable variation regularization.

EDITED REFERENCES

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