Wavefield reconstruction using a Stolt-based asymptote and apex shifted hyperbolic Radon transform

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SUMMARY

The asymptote and apex shifted hyperbolic Radon transform is an extension to the apex shifted hyperbolic Radon transform allowing shifts in both the asymptote and apex of hyperbolic events. Thanks to the additional degree of freedom, the transform can better focus seismic reflections and diffractions in common shot domain. An efficient transform kernel is implemented using Stolt migration and demigration operators and applied to the interpolation of a simple synthetic dataset and a common shot gather from the 2D BP/SEG salt model.

INTRODUCTION

In towed-streamer marine seismic acquisition, operational costs and entanglement issues pose substantial limits to sampling along the spatial crossline axis. This can lead to aliasing problems and lower the imaging resolution of the subsurface. Therefore, seismic data usually require interpolation to increase the spatial sampling prior to time-domain processing and imaging.

Most interpolation algorithms rely on transforms that can focus seismic data in the transform domain. These focusing abilities arise from the similarity between seismic data and the chosen transform dictionary. This justifies the ongoing research into new dictionaries that closely match seismic data for interpolation applications (Terenghi, 2014).

Since the travel-times of a variety of seismic events can be approximated by hyperbolas, Radon transforms that use a hyperbolic dictionary represent a powerful tool for interpolation (Sacchi and Ulrych, 1995; Trad, 2003). The most common of these transforms is the Hyperbolic Radon Transform (HRT), which is used for processing common midpoint gathers, where the apexes of seismic reflection hyperbolas are usually located at zero offset (Thorson and Claerbout, 1985). However, Trad (2003) proposed interpolating seismic data in the common shot gather domain using an Apex Shifted Hyperbolic Radon Transform (ASHRT), which extends the conventional HRT by scanning for the horizontal location of apexes. The ASHRT has been applied in order to interpolate and/or denoise seismic data in the shot gather domain (Trad et al., 2003; Ibrahim and Sacchi, 2014c) or as part of 3D multiple prediction algorithms (van Dedem and Verschuur, 2000, 2005).

In this paper, we present an additional extension to the ASHRT by scanning for both the apex and asymptote shifts to match both reflection and diffraction hyperbolas more closely. Seismic diffractions can provide important information about subsurface discontinuities such as faults, pinch outs and small size scattering objects that can be used in interpretation or imaging (Khaidukov et al., 2004; Bansal and Imhof, 2005; Klokov and Fomel, 2012). The travel time curve of diffracted waves can be represented by the double square root equation (Landa et al.,

1987; Kanasewich and Phadke, 1988)

$$t = \sqrt{t_d^2 + (x_s - x_d)^2/v^2} + \sqrt{t_d^2 + (x_d - x_r)^2/v^2} \quad (1)$$

where t_d is the one way travel time for the diffraction, x_d is the diffraction location along the horizontal axis, v is the velocity at the diffraction, x_s is the source location and x_r is the receiver location. If we are considering a common shot gather, the value of the first square root is constant for each diffraction hyperbola. Therefore, we introduce the new parameter, $\tau_0 = \sqrt{t_d^2 + (x_s - x_d)^2/v^2}$, into the previous equation,

$$t = \tau_0 + \sqrt{t_d^2 + \frac{(x_d - x_r)^2}{v^2}}. \quad (2)$$

We use this equation to define the new Asymptote and Apex Shifted Hyperbolic Radon Transform (AASHRT) which scans for the asymptote origin time shift τ_0. This equation simplifies to the ASHRT definition when the asymptote origin time shift τ_0 is set to zero and the diffraction time t_d and location x_d are replaced by the apex time τ and location x_a, respectively.

An efficient implementation of the AASHRT can be obtained using Stolt migration/demigration operators (Stolt, 1978; Trad, 2003; Ibrahim and Sacchi, 2014b,a). The asymptote shift introduced by the AASHRT can be incorporated easily into the Stolt kernel using the Fourier time shift property. In case of uniform sampling, computational efficiency can be further increased using fast Fourier transforms and by scanning only the non zeros elements of the data in the $\omega - k$ domain.

ASYMPTOTE AND APEX SHIFTED HYPERBOLIC RADON TRANSFORM

The AASHRT models seismic data using a superposition of asymptote and apex shifted hyperbolas as follows

$$d(t, x_r) = \sum_{\tau_0} \sum_{x_a} \sum_{v} m(\tau = \sqrt{t^2 - \frac{(x_r - x_a)^2}{v^2}} - \tau_0, v, x_a, \tau_0)$$

$$(3)$$

where $d(t, x_r)$ is the modelled seismic data and $m(\tau, v, x_a, \tau_0)$ is the AASHRT model. A low resolution AASHRT model can be estimated using the adjoint operation as follows

$$\widetilde{m}(\tau, v, x_a, \tau_0) = \sum_{x_r} d(t = \tau_0 + \sqrt{\tau^2 + \frac{(x_r - x_a)^2}{v^2}}, x_r) \quad (4)$$

where $\widetilde{m}(\tau, v, x_a, \tau_0)$ is the estimated AASHRT model. These transforms can be rewritten in the operator format as

$$\mathbf{d} = \mathbf{Lm}, \quad (5)$$

$$\widetilde{\mathbf{m}} = \mathbf{L}^T \mathbf{d}, \quad (6)$$

where $\mathbf{d}$, $\mathbf{m}$ and $\widetilde{\mathbf{m}}$ represent the data, model and estimated model in vector form, respectively. The forward and adjoint AASHRT operators are represented by $\mathbf{L}$ and $\mathbf{L}^T$, respectively.

STOLT BASED AASHRT

The time domain implementation of the AASHRT operator is computationally intensive. Fortunately, the AASHRT kernel can be computed efficiently in the $\omega - k$ domain using fast Stolt migration/demigration operators. These operators perform migration by mapping data in the $\omega - k_x$ domain into $\omega_\tau - k_x$ for a constant subsurface velocity using the dispersion relation (Yilmaz, 2001; Ibrahim and Sacchi, 2014b,a)

$$\omega_\tau = \sqrt{\omega^2 - (vk_x)^2} \qquad (7)$$

where ω_τ is the Fourier dual of the apex time τ, k_x is the horizontal wavenumber and v is the subsurface velocity. Using the exploding reflector principle (Claerbout, 1992) and the constant subsurface velocity assumption, the Stolt migration operator can be used to estimate the subsurface model. Similarly, the Stolt migration operator can be used to estimate the AASHRT model $\tilde{m}(\tau, v, x_a, \tau_0)$ as follows

$$\tilde{m}(\tau, v, x_a, \tau_0) = C \int \int \exp[i\omega_\tau \tau_0] d(\omega = \sqrt{\omega_\tau^2 + (vk_x)^2}, k_x)$$
$$\times \exp[-ik_x x - i\omega_\tau(v)\tau] \, d\omega_\tau \, dk_x \qquad (8)$$

where $C = v \, (\omega_\tau/\omega)$ is a scaling factor resulting from the change of variables. It is worth emphasizing that, using Stolt-based operators, (1) every horizontal axis location x is treated as a possible receiver and apex location (so $x_a \equiv x_r \equiv x$) and (2) the periodicity of $\tilde{m}$ along x is implied.

Stolt-based forward modelling can be written as

$$d(t, x) = \int \int \int \int m(\omega_\tau = \sqrt{\omega^2 - (vk_x)^2}, v, k_x)$$
$$\times \exp[-i\omega_\tau \tau_0] \exp[ik_x x + i\omega t] \, d\omega \, dk_x \, dv \, d\tau_0 \quad (9)$$

The adjoint transforms in equation (9) can be rewritten in operator form as

$$\mathbf{L}^T = \mathbf{FFT}_{\omega_\tau, k_x}^{-1} \, \mathbf{M}_{\omega, v, k_x}^T \, \mathbf{T}^T \, \mathbf{FFT}_{t,x} \, \mathbf{A}^T \mathbf{S}^T, \qquad (10)$$

and similarly the forward (modelling) operator (equation 8) can be written as

$$\mathbf{L} = \mathbf{S} \, \mathbf{A} \, \mathbf{FFT}_{\omega, k_x}^{-1} \, \mathbf{T} \, \mathbf{M}_{\omega_\tau, v, k_x} \, \mathbf{FFT}_{\tau, x}, \qquad (11)$$

where, $\mathbf{FFT}$ is the Fast Fourier Transform, $\mathbf{M}_{\omega_\tau, v, k_x}$ is the Stolt mapping operator, $\mathbf{A}$ is a summation operator and its adjoint is a spraying operator (Claerbout, 1992). Operator $\mathbf{T}$ represents the time shift operator in frequency domain while $\mathbf{S}$ represents the sampling operator used for interpolation (Liu and Sacchi, 2004). In case of non-uniform spatial sampling, the $\mathbf{FFT}$ operator may be replaced by a Discrete Fourier Transform operator or a non-uniform Fast Fourier Transform operator.

SPARSE INVERSION

The estimated model $\tilde{\mathbf{m}}$ and the original model $\mathbf{m}$ are clearly not identical because Radon transforms are not orthogonal transformations ($\mathbf{LL}^T \neq 1$). Furthermore, seismic reflection data may be affected by a number of disturbing factors, including significant noise, limited spatial aperture, coarse and irregular spatial sampling, missing traces, etc. The estimation of the Radon model must then be posed as an inversion problem (Thorson and Claerbout, 1985) conditioned by a regularization (penalty) term. The general form of the cost function to be minimized in order to obtain the Radon coefficients is given by (Ibrahim and Sacchi, 2014c)

$$J = \|\mathbf{d} - \mathbf{L}\mathbf{m}\|_p^p + \mu \|\mathbf{m}\|_q^q \qquad (12)$$

where μ is the trade-off parameter that controls the relative weight between the model regularization term $\|\mathbf{m}\|_q^q$ and the misfit term $\|\mathbf{d} - \mathbf{L}\mathbf{m}\|_p^p$. Furthermore, parameters p and q indicate the order of the norms used for the misfit and regularization terms. Since we choose a Radon dictionary that closely matches the seismic data, the Radon coefficients should be sparse. We therefore choose an ℓ_1 norm ($q = 1$) for the model regularization term and an ℓ_2 norm ($p = 2$) for data misfit term. This cost function can be minimized using the Fast Iterative Shrinkage/Threshold Algorithm (FISTA) (Beck and Teboulle, 2009). This algorithm requires an approximation for the largest eigenvalue of the $\mathbf{L}^T\mathbf{L}$ operator which is calculated using Rayleigh's power method (Larson and Edwards, 2009). For details about the FISTA algorthim and its application in geophysics refer to Rodriguez et al. (2012) and Perez et al. (2013).

EXAMPLES

We evaluate the proposed AASHRT interpolation algorithm on a set of purpose-built synthetics composed of well distanced hyperbolic events, including 3 reflected and 3 diffracted events. Each reflection represents the acoustic response of a planar interface overlaid by a constant velocity medium. The interfaces, characterized by dips of 10, 20 and -30 degrees, are located at depths of 300, 600 and 900m below the source, respectively. The overlaying media are characterized by propagation velocities of 1500, 1750 and 2000 m/s. The 3 diffractions originate from features located at a common depth of 400m and horizontal offsets of -500, 0, and 1500m with respect to the source. In its undecimated version the test dataset is uniformly sampled with traces 12.5m apart (figure 1b), to ensure absence of spatial aliasing up to 60Hz (figure 1f). The subset used as input to the interpolation algorithm is obtained by uniformly under-sampling the initial data by a factor of 5 (one in every five traces is kept). The algorithm, parametrized to recover data at the initial spatial sampling rate of 12.5m produces the results shown in figure 1.

We also tested the new transform on synthetics modelled using the 2004 BP velocity benchmark (Billette and Brandsberg-Dahl, 2005). Similar to the previous example, the data in its undecimated version is uniformly sampled with traces 12.5m apart. The input to the interpolation algorithm is obtained by uniformly under-sampling the initial data by a factor of 3 (one in every three traces is kept). We recover the data at sampling rate of 12.5 m producing the results shown in figure 2.
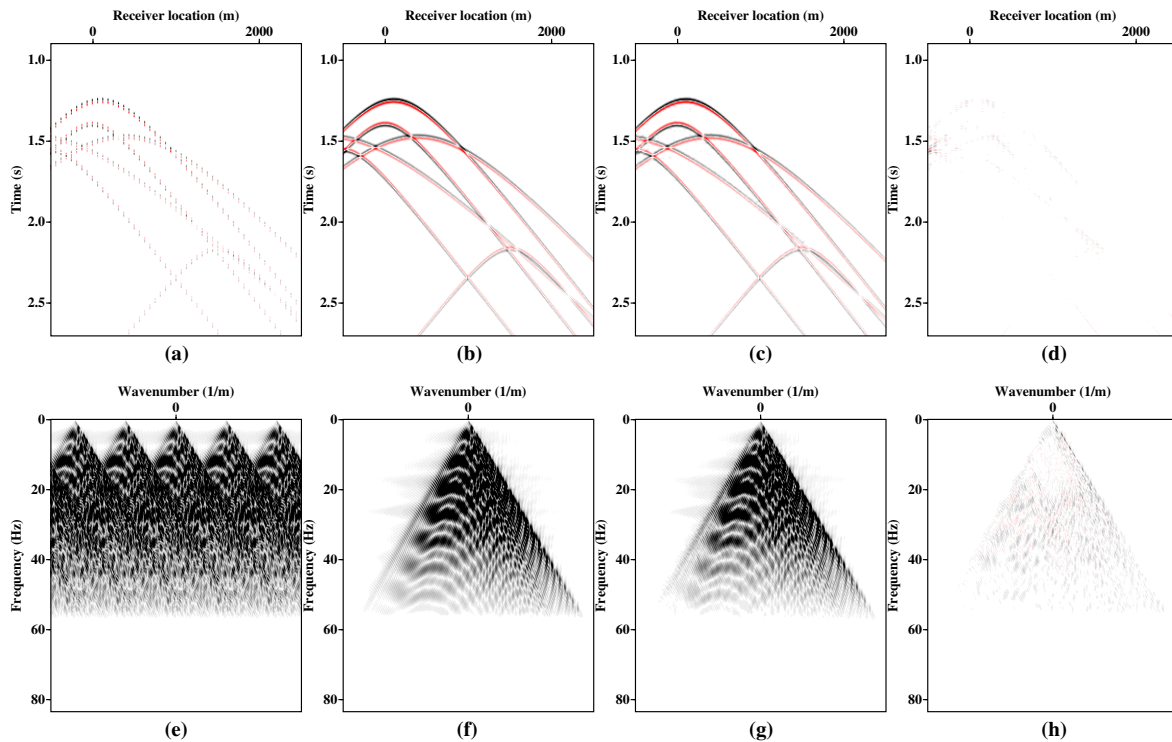
Figure 1: Simplified synthetic shot gather example. (a) Decimated gather. (b) Original gather. (c) Interpolated gather. (d) Interpolation error. (e), (f), (g) and (h) are the $\omega - k_x$ spectra of (a), (b), (c), and (d), respectively.

The quality of the interpolation is calculated using the following formula:

$$Q = 10 \, log_{10} \left(\frac{\|\mathbf{d}_{original}\|_2^2}{\|\mathbf{d}_{original} - \mathbf{d}_{recovered}\|_2^2} \right). \tag{13}$$

The Q value for the recovered simple synthetic shot gather is 17.09 dB and for the BP/SEG model shot gather is 14.99 dB.

CONCLUSION

We have implemented an asymptote and apex shifted hyperbolic Radon transform with a Stolt migration/demigration operator as its kernel to speed up computation. The new transform dictionary is designed to closely match both reflections and diffractions. Our tests show that the new transform is a suitable tool for interpolation. Since the new transform is implemented in $\omega - k$ domain, it can be used in combination with the non-uniform Fourier transform to improve interpolation. Future work entails generalizing the problem to the 3D shot distribution.
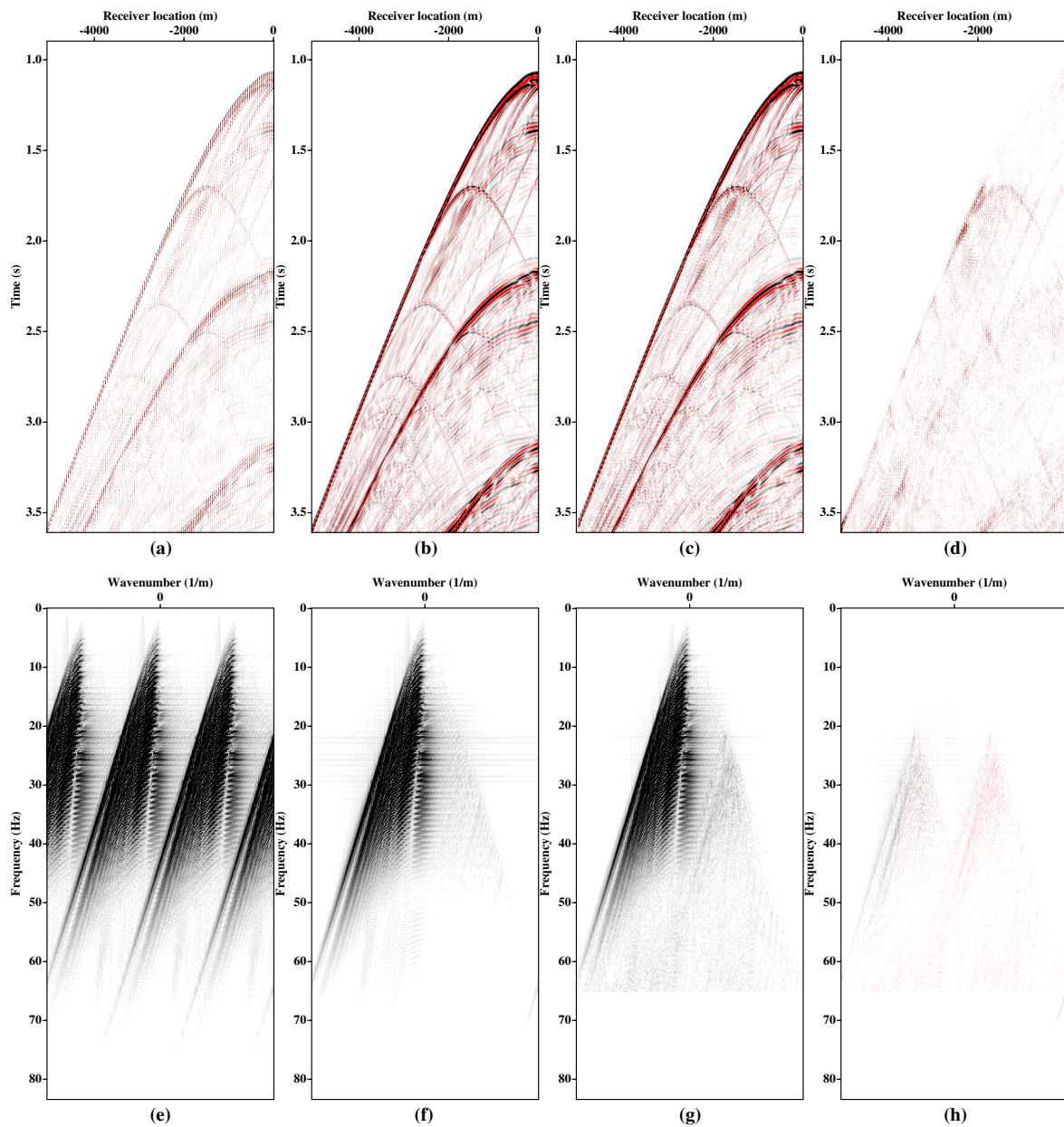
ACKNOWLEDGEMENTS

Figure 2: Shot gather example of the BP/SEG velocity model. (a) Decimated gather. (b) Original gather. (c) Interpolated gather. (d) Interpolation error. (e), (f), (g) and (h) are the $\omega - k_x$ spectra of (a), (b), (c), and (d), respectively.

EDITED REFERENCES

Note: This reference list is a copyedited version of the reference list submitted by the author. Reference lists for the 2015 SEG Technical Program Expanded Abstracts have been copyedited so that references provided with the online metadata for each paper will achieve a high degree of linking to cited sources that appear on the Web.

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