A NEW LOOK AT AUTOREGRESSIVE LOW FREQUENCY RECONSTRUCTION OF SEISMIC DATA

M. Bekara[1], R. Djebbi[1], N. Chemingui[1]

[1] PGS

Summary

This abstract proposes a novel, easy to deploy and computationally fast solution to reconstruct the low frequency content of seismic data for FWI application. This solution is needed when the SNR is very poor or when denoise fails to enhance sufficiently the low frequency content of the data. The method follows a signal processing approach and does not need to train an offline model on auxiliary data, as is the case for solutions based on machine learning. The reconstruction is done from the higher frequencies using a recursion filter which is estimated from the data itself. The novelty of the proposed method is that it transforms the data locally from the time-space domain to the time-slowness domain where the reconstruction is performed. This transformation enforces the time domain sparsity needed to justify the use of recursion modeling in the frequency domain and exploits the spatial coherency in the data. The method also implicitly uses the signal cone limits of the seismic wavefield to provide a physically constrained solution. The proposed method was tested on many datasets to condition it for FWI and proved to be successful to help the inversion to mitigate cycle skipping and to improve the final model.

A new look at autoregressive low frequency reconstruction of seismic data

Introduction

The presence of low frequency content in seismic data is very important for subsurface imaging. In full wave inversion (FWI), low frequencies provide protection against cycle skipping and ensure convergence to the right velocity model. They are key to estimate the correct background of the model which influences the subsequent velocity updates, as higher frequencies are included in the inversion. Post imaging applications such as acoustic impedance inversion needs low frequencies to estimate the trend properly. In the absence of well-log information, they are derived from the seismic data. Despite the massive improvements in seismic acquisition and receiver technology, it is often difficult to get low frequency content (< 4 Hz) with good signal-to-noise ratio (SNR). This is mainly due the presence of strong low frequency ambient noise and/or to physical limitations of the source's power. Noise attenuation is routinely used to boost the SNR; however this solution fails when the input SNR is very poor. The alternative option is to reconstruct the valuable low frequencies.

Early work on low frequency reconstruction dates back to the 1980s. It was proposed to improve the result of acoustic impedance inversion from post-stack data (Walker and Ulrych, 1983). The method used an autoregressive model, derived from higher frequencies, to fill-in the missing low frequencies. It was not until the emergence of FWI, as a standard imaging tool, that work on this topic became of interest again. Wu, et al. (2014) used the slow varying envelope of the seismic traces in conjunction with the FWI formulation to estimate the low wavenumbers of the velocity model. Li and Demanet (2016) proposed to decompose the data into frequency-dependent amplitude & phase atoms and then assumed respectively a linear and constant extrapolation model for the low frequencies. Wang and Herrmann (2016) inverted the low frequencies by minimizing an L_1cost function on the traces in the time domain with a total-variation regularization that accounts for spatial correlation between traces.

Machine learning (ML) gave a boost and a new direction to the effort of low frequency reconstruction. The literature contains many papers on this technology, such as the early work by Ovcharenko et al. (2017) and the more recent one by Sun et al. (2021). All the contributions differ mainly in the type of the network's architecture used in the learning process but share one common aspect. Synthetic data with reliable low frequency contents are generated, using fine-difference modeling, and then the high frequencies are used as an input to train the selected network to predict low frequencies. ML is practical as long as the generalization of the network is good and robust. The validation data examples shown in the literature, using this technology, were predominately synthetic and generated under a similar geology and acquisition parameters to the dataset used in the training process. We believe that a field deployment of this technology on real data would require constant re-training for every new project to improve the robustness. This will not only make the solution expensive but may bias the FWI results toward the velocity field used in generating the training data.

In this abstract we propose a computationally affordable, easy to use method that belongs to the signal processing toolbox to reconstruct the low frequency contents of seismic data for various applications.

Proposed method

The proposed method is adapted from the work of Walker and Ulrych (1983) and is based on the idea that a sequence of spikes in the time domain maps into a superposition of linear harmonics in the frequency domain which can accurately be modelled by an autoregressive filter. Deviation from pure sparsity in the time domain can be caused by background reflections, noise and bandlimited source and this would translate into distortion of the harmonic assumption. In the proposed method we try to limit the impact of this deviation. To reconstruct the data below a cut-off frequency $\boldsymbol{f_c}$, the proposed method works on a sliding time-space window and performs the following steps:

1. Map the data from $(\boldsymbol{t}-\boldsymbol{x})$ to $(\boldsymbol{f}-\boldsymbol{p_x})$ (frequency-slowness domain)
2. Loop over $\boldsymbol{p_x}$,
    - Set the frequency $\boldsymbol{F}=\boldsymbol{f_c}$

- While $\boldsymbol{F} \geq \mathbf{0}$, Do
  2.1 Fit a linear/nonlinear recursion formula to the complex sequence in $[\boldsymbol{F}, \boldsymbol{F}+\boldsymbol{B}]$, where $\boldsymbol{B}$ is the reconstruction bandwidth
  2.2 Predict the data at $\boldsymbol{f} = \boldsymbol{F} - \Delta \boldsymbol{f}$

3. Set $\boldsymbol{F} = \boldsymbol{F} - \Delta \boldsymbol{f}$
4. Map the data from $(\boldsymbol{f} - \boldsymbol{p_x})$ to $(\boldsymbol{t} - \boldsymbol{x})$

The algorithm above is different from the one proposed by Walker and Ulrych (1983) in three different aspects. First, the reconstruction is applied over a local window in time-space rather than on the entire trace to adapt to the non-stationarity of the seismic data. Second, the reconstruction is done in the slowness-frequency domain rather than in the frequency domain to enforce sparsity assumption, to include the spatial correlation in the data and to honour the constraint of the "signal cone" of the wavefield. Third, the reconstruction is done stepwise, i.e., a frequency sample is reconstructed each time with a re-estimation of the recursion model. This offers some protection against deviation of the model from the linear harmonic assumption.

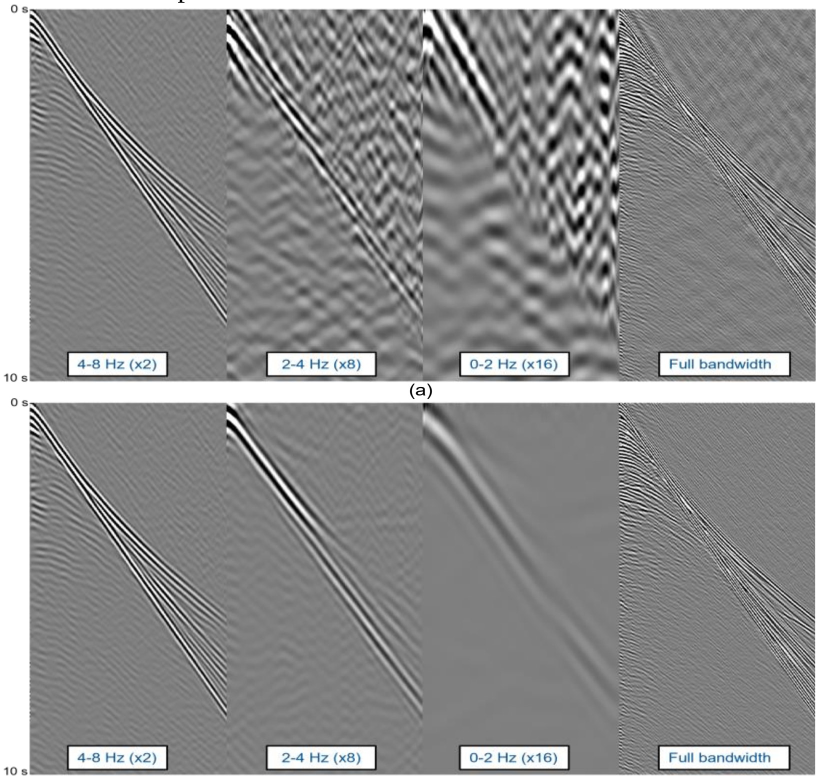

Figure 1 *Shot gather with its frequency panels. (a) Input, (b) after low frequency reconstruction below 4Hz*

Field data example

We apply the proposed method to re-build the low frequency contents of a marine seismic data example before running FWI as an alternative data conditioning to conventional denoise. The data is a 2D towed streamer line acquired offshore Malaysia using the continuous wavefields method (Klüver et al., 2020). The streamer is dual sensor with a maximum inline offset of 8.1 km. The water bottom is shallow and ranges between 125 m to 200 m. Figure 1-a shows a single shot gather (full band) and some of its lower frequency panels. The SNR in the data is very poor below 4 Hz due to background noise and contamination from other sources. Normally, this band of the data is ignored when running FWI. However, this will prevent us from the opportunity to avoid cycle skipping when the initial model is not accurate. We have applied the proposed method, in the shot domain, to reconstruct the signal below 4 Hz and the result is shown in Figure 1-b. In the band 2-4 Hz, the process did an excellent job in cleaning the data and clearly achieved a sensible reconstruction below 2 Hz. Inspecting the FK spectrum of the shot before and after the low frequency reconstruction in Figure 3, we can see that the water bottom reflection and refraction are nicely extended below 4 Hz (arrow in Figure 2-b). The noise (arrows in Figure 2-a) is nicely removed without the imprint of a surgical FK filter. We can appreciate, in addition to the reconstruction ability of the proposed method, its excellent denoising effect. To assess the phase of the reconstructed data and its consistency with the higher frequencies, we display in Figure 3 portions of the shot interleaved between the input original data (below 6 Hz) and the reconstructed data (below 4 Hz). This is a good display QC to assess the phase of the extrapolated data at different offsets. We can clearly see a very good continuity of all the linear events across the different offsets which gives us confidence that the low frequency extrapolation is done in a consistent manner that will help FWI.

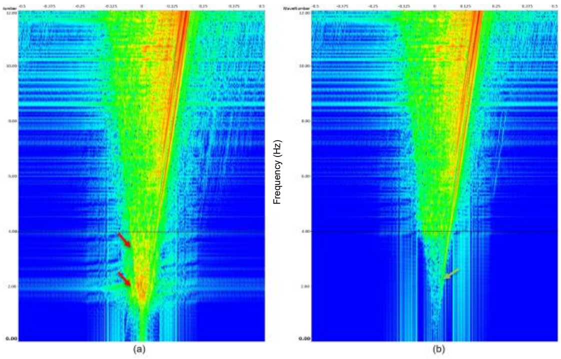

Figure 2 *FK transform of the shot, (a) Input (b) after low frequency reconstruction*

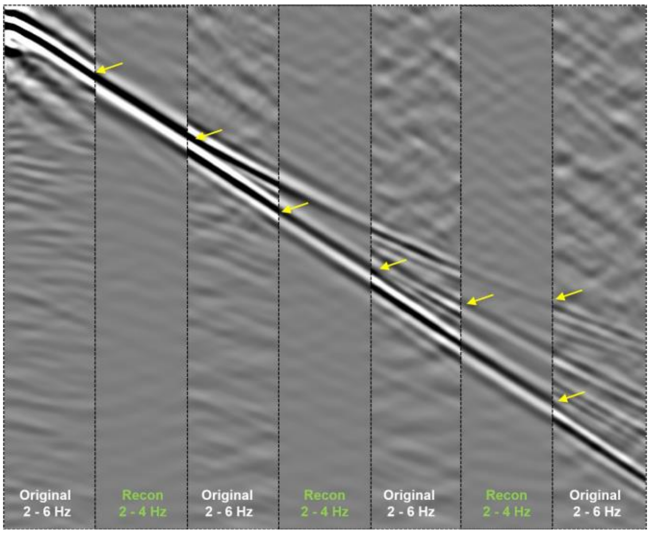

Figure 3 *Interleaved band-passed input and reconstructed data (< 4Hz) throughout the shot*

To assess the quality of the results on many shots, the low frequency reconstruction is run for the entire line. Figure 4 shows a 2D stack (full band) and two frequency panels up to 4 Hz. The stack is extremely noisy below 2 Hz and very noisy between 2 Hz to 4 Hz. The process was pretty effective in enhancing the low frequency structure (< 2 Hz) near the water bottom and providing an effective denoise for the 2-4 Hz band. To see if the low frequency reconstruction does indeed help FWI, we run two FWI tests with and without low frequency reconstruction (< 4Hz). The initial velocity model is a constant gradient model (Figure 5-a). Each FWI test consists of seven cascaded frequency stages up to 16 Hz (1-2 Hz; 1-3 Hz; 2-4 Hz; 2-6 Hz; 2-8 Hz; 2-12 Hz; 2-16 Hz), where the output of each stage is fed as an initial model to the next one. For the test where no frequency extrapolation is performed, the first three stages were skipped. The final FWI results are show in Figure 5-b,c and there is a visible difference between the two models. To QC the data fitting of each model, we compare the field data and the modeled data using each of the models. With the initial model, there is a severe cycle-skipping in mid to far offset as the model is trying to account for both the shallow reflections and the refraction. After FWI with no low frequency extrapolation, the cycle-skipping is mainly in the refracted energy which is wrongly aligned with the reflections in many places because of the wrong updates in the shallow part of the model. The FWI results with low frequency extrapolation is almost perfect and the transition between field and modeled data is seamless. The proposed method has been applied on many datasets and shows the same added benefit for FWI (Djebbi et al., 2022).

Conclusion

We have proposed a novel, easy to deploy and computationally fast solution to reconstruct the low frequency content of seismic data. The method does not need any costly offline training and follows a conventional signal processing approach. It works robustly below 4 to 5 Hz, although it can functionally reconstruct any frequency range. This solution has been applied to many datasets to condition them for FWI and proves to be successful in helping the inversion to mitigate cycle skipping. The proposed solution has the potential to change the best practice in FWI by allowing the use of the low frequencies that are often ignored due to poor SNR. Finally, the method can also be used as a generic low frequency denoise tool.

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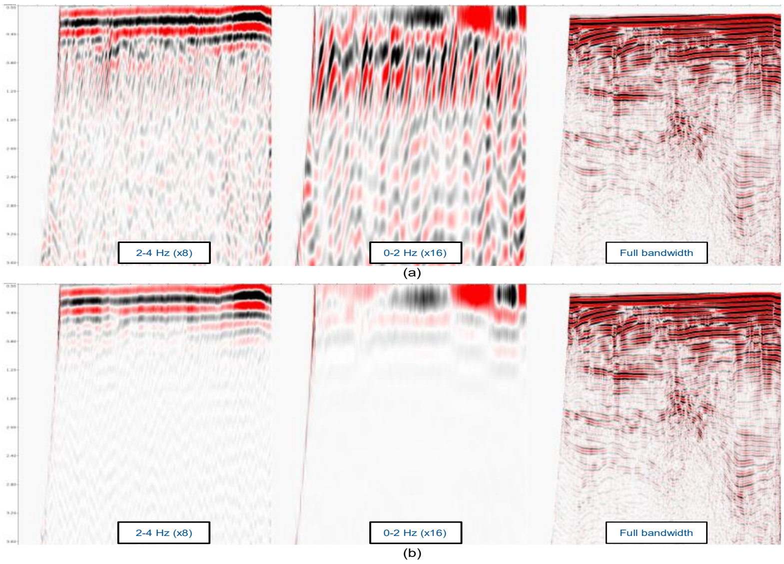

Figure 4 *2D stack with its frequency panels. (a) Input, (b) after low frequency reconstruction below 4Hz*

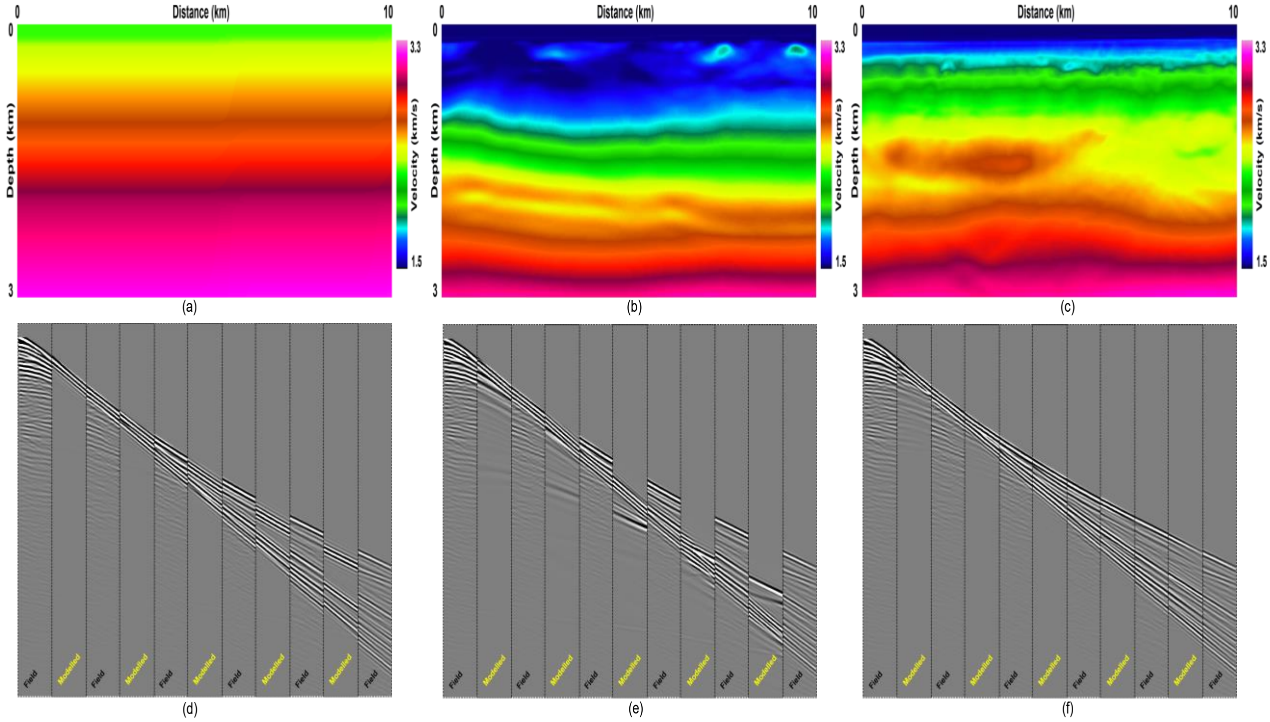

Figure 5 *(a) Initial model, (b) inverted model with no low frequency reconstruction (LFR), (c) inverted model with LFR. Model fitting QC using: (d) initial model (e) model with no LFR, (f) model with LFR*

References

Djebbi, R., Bekara, M., Chemingui, N., and Ramos-Martínez, J. [2020] Full waveform inversion with low frequency reconstructed data. Submitted to 83rd Annual Int. Conference and Exhibition, EAGE

Klüver, T., Hegna, S., and Lima, J. [2020] Continuous wavefields method: Insights from a shallow water field trial. SEG Expanded Abstracts

Li, Y. E., and Demanet, L. [2016] Full-waveform inversion with extrapolated low-frequency data. Geophysics, **81**, R339-R348.

Sun, H., and Demanet, L. [2021] Deep Learning for Low Frequency Extrapolation of Multicomponent Data in Elastic FWI. IEEE Transactions on Geoscience and Remote Sensing

Walker, C., and Ulrych, T. J. [1983] Autoregressive recovery of the acoustic impedance Geophysics. **48**, 1338–1350

Wu, R.S., Luo, J., and Wu, B. [2014] Seismic envelope inversion and modulation signal model. Geophysics, **79**,WA13-WA24

Wang, R., and Herrmann, F. [2016] Frequency down extrapolation with tv norm minimization. SEG Expanded Abstracts