MIXED PHASE SEISMIC WAVELET ESTIMATION USING THE BISPECTRUM

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Summary

The ability to estimate a mixed phase wavelet is a useful tool for processing and quality control in seismic imaging. The wavelet is estimated using higher order statistics of the data. In practice, these methods tend to show some instability issues when the wavelet length is increased. To improve the stability of the solution, this abstract proposes a new formulation of the wavelet estimation problem that constrains the solution to be a finite duration, phase-only compensation applied to a known base wavelet. The proposed solution works in the frequency domain and consists of three steps. First, the bispectrum of the data is deconvolved using the bispectrum of the base wavelet to increase its bandwidth. This helps to improve the sensitivity of third order statistics to phase information. Then, a phase-only wavelet is estimated from the deconvolved bispectrum using an iterative least-squares approach without phase unwrapping. Finally, the estimated phase-only wavelet is conditioned using a projection onto convex sets type algorithm to enforce the constraint of the finite time duration giving the user a control on the amount of phase deviation from the base wavelet. Test examples on synthetic and real data both show reliable results with robustness to noise contamination.

Introduction

Knowledge of the wavelet and more particularly its phase are key elements in the processing of seismic data. In marine seismic processing, the wavelet is closely controlled through a set of deterministic processes such as source/receiver deghosting, debubbling, designature and Q-compensation. Despite this control, phase compensation is often needed for deeper reflectors mainly due to the approximations in the physics behind these deterministic processes. In the absence of well-log data, estimating the wavelet from seismic data is important to perform this compensation. Moreover, the estimated wavelet can be used for quality control (QC) to assess the phase integrity of the seismic data as it progresses through the processing sequence.

This abstract proposes a novel statistical method to estimate a mixed phase wavelet from seismic data. The method derives the amplitude spectrum of the wavelet using a smooth log-spectral estimator and uses higher order statistics to extract the phase information. Contrary to similar methods that use cumulant matching, the proposed method determines the phase of the wavelet from a deconvolved *bispectrum* of the data using a least-squares solver and without phase unwrapping. The motivation to bypass time domain cumulant matching is to avoid solving a complex optimization problem with many local minima. Even when using stochastic optimizers, the solution tends to be unstable when the wavelet length is large (> 50 samples) or when the signal-to-noise ratio is moderate to low. The method also avoids phase unwrapping as this process is highly sensitive to the presence of noise and breaks down when the phase profile is not smooth. The proposed method is demonstrated on both synthetic and real data and shows consistently robust results with regards to the noise contamination and the length of the wavelet.

Wavelet estimation using higher order statistics

Most of the statistical wavelet estimation methods assume the following convolution model:

$$d_n(t) = r_n(t) * w(t) + \varepsilon_n(t) \ \ , n = 1,2,\cdots N, \qquad (1)$$

where $d_n(t)$ is the seismic trace with index n, $r_n(t)$ is the corresponding reflectivity series, $w(t)$ is the desired wavelet and $\varepsilon_n(t)$ is a random noise term. Equation (1) is valid only post-stack and post-migration with an appropriate Q (phase and amplitude) compensation applied to the data. A large number of these methods are based upon second-order statistics (i.e., autocorrelation) and make use of a minimum phase or zero phase assumption. Since the mid-1980s a number of authors have used higher order statistics to solve the general problem of blind system identification in signal processing. These methods relax the assumption on the phase of the system but bring an additional requirement that the system input (i.e. excitation) is an independent and identically distributed non-Gaussian random process. In our context, this means that the reflectivity series is sparse with a white spectrum. Many approaches have been developed for system identification using both third and fourth order statistics in time (cumulant) and frequency (polyspectral) domains. A review of these methods can be found in Mendel (1991).

In seismic data processing, cumulant matching techniques are the first set of higher order statistical methods to be practically adopted for wavelet estimation and they are considered the reference in this matter (Lazear, 1993). For simplicity, considering the case of a third order cumulant, these methods find the wavelet $w(t)$ whose third order cumulant matches that of the data in a least-squares sense, i.e., by minimizing the following cost function

$$\min_{w}\left\{\left(\sum_{\tau_1}\sum_{\tau_2}\left[C_d^3\left(\tau_{1,}\tau_2\right) - \sum_{m} w(m)w(m-\tau_1)w(m-\tau_2)\right]^2\right)\right\} \qquad (2)$$

where $C_d^3(\tau_1,\tau_1)$ is the aggregate cumulant obtained by tapering and weighted stacking of all the third order cumulants $C_{d_n}^3(\tau_1,\tau_1)$ of individual traces $d_n(t)$, defined as:

$$C_{d_n}^3(\tau_1,\tau_1) = \sum_{t} d_n(t)d_n(t-\tau_1)d_n(t-\tau_2) \qquad (3)$$

The minimization problem in equation (2) is non-convex and highly nonlinear with many local minima. Any gradient descent solution would need an accurate initial guess otherwise it will get stuck in a local minimum. Velis and Ulrych (1996) proposed a stochastic global optimizer based on fast simulated annealing to solve this problem. However, in all their examples the length of the wavelet did not exceed

40 samples (160 ms). Cumulant matching methods tend to produce unstable results when the number of samples to estimate increases. Other sets of methods, less known in the seismic processing industry, use the bispectrum to estimate the phase of the wavelet (Matsuoka and Ulrych, 1984). These methods work in the frequency domain and are not constrained by the wavelet length. They are based on the following relationship that exists between the bispectrum of a trace $x(t)$ and its Fourier transform (for third order statistics):

$$\Gamma_x^3(f_1, f_2) = FFT2D\{C_x^3(\tau_1, \tau_2)\} = X(f_1)X(f_2)X^*(f_1 + f_2) \qquad (4)$$

Using the above equation, the phase of the wavelet φ_w is related to the phase of the bispectrum of the data φ_d as:

$$\varphi_d(f_1, f_2) = \varphi_w(f_1) + \varphi_w(f_1) - \varphi_w(f_1 + f_2) \qquad (5)$$

Equation (5) is solved using least-squares after phase unwrapping (Zhang et al., 2009). Phase unwrapping is a difficult process, particularly for 2D complex signals, and is very sensitive to the level of noise in the data (Ghiglia and Pritt, 1998). This is the main reason the bispectrum is not practically used for seismic wavelet estimation.

Proposed method

A new formulation is proposed to estimate the seismic wavelet $w(t)$ as a non-parametric phase-only perturbation to a known base wavelet $w_0(t)$, i.e.:

$$\begin{aligned} &w(t) = w_0(t) * h(t) \\ with\ \ &h(t) = 0 \quad |t| > T \\ and\ \ &|H(f)| = 1 \end{aligned} \qquad (6)$$

The problem simplifies to the estimation of a phase-only wavelet $h(t)$. The *gap time* (T) controls how much one wants to deviate from the base wavelet. The larger the gap value, the more phase compensation one can apply to $w_0(t)$. The solution is developed for the case of third order statistics, but generalisation to fourth order is straightforward. It consists of the following three steps:

1. *Deconvolution*

Using equations (1) and (6) one can relate the bispectrum of the data with that of $w_0(t)$ and $h(t)$. The bispectrum of $h(t)$ is estimated in the following least-squares sense:

$$\min_{\Gamma_h^3(f_1,f_2)} \sum_{n=1}^{N} \left|\Gamma_{d_n}^3(f_1, f_2) - \Gamma_{w_0}^3(f_1, f_2)\Gamma_h^3(f_1, f_2)\right|^2 \qquad (7)$$

The solution to the optimisation problem is the classical Weiner deconvolution filter, i.e.:

$$\Gamma_h^3(f_1, f_2) = \frac{1/_N \sum_{n=1}^{N} \Gamma_{d_n}^3(f_1,f_2)\Gamma_{w_0}^3(f_1,f_2)^*}{\left|\Gamma_{w_0}^3(f_1,f_2)\right|^2 + \epsilon} \qquad (8)$$

The deconvolution plays the role of a whitening process that increases the bandwidth of the data and makes the higher order statistics more sensitive to phase information. $\Gamma_h^3(f_1, f_2)$ is then normalised and denoted by $\bar{\Gamma}_h^3(f_1, f_2)$.

2. *Phase estimation*

Using equation (4) and rather than performing phase unwrapping, we solve for $H(f)$ directly with an iterative least-squares approach. Figure 1 shows the flowchart for the phase estimation. The process is iterative with a stable convergence property as tested on synthetic and real data sets. The initial solution does not affect the final one and we have found that a zero phase initial solution always gives good and stable results.

3. *Phase conditioning*

This step is needed to enforce the constraint on $h(t)$ of being of limited time support. Figure 2a shows the flowchart of the algorithm, which belongs to the family of iterative projection methods, known as Projection onto convex sets (POCS) (Figure 2b). This algorithm is proven to converge to a solution that satisfies the two constraints in equation (6).

Data examples

1. *Synthetic data*

Figure 3 shows a synthetic reflectivity section that contains three reflectors and the resulting seismic

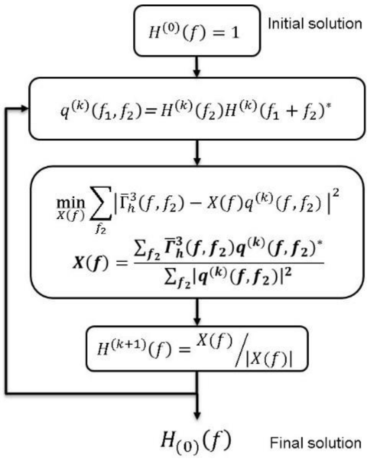

Figure 1. *Flowchart for the phase estimation algorithm.*

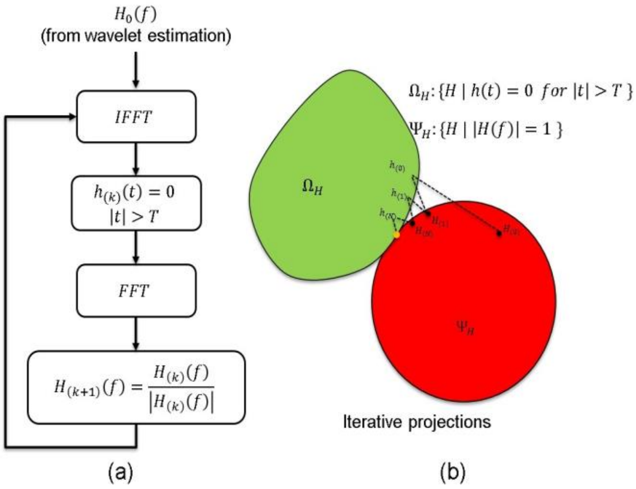

Figure 2. *Flowchart for the phase conditioning algorithm (a), geometrical illustration of the algorithm (b)*

data (680 ms length @ 4 ms sampling) after convolving it with a wavelet of length 300 ms and adding some synthetic random noise. The estimated wavelet using the suggested method with $\boldsymbol{T} = 200$ ms and a zero phase base wavelet is shown in Figure 4. It compares closely to the true wavelet and its phase spectrum is unbiased and fluctuates around the true phase. Using a minimum phase base wavelet would give the same results. The amplitude spectrum is accurate and non-jittery, leading to a smooth looking wavelet.

2. *Real data*

Figure 5a shows a 1000 ms window extracted from a central subline section of a final PSTM volume. A cube of 201x201 traces centred on the window are used to estimate a 500 ms length wavelet. The proposed method is tested with a zero phase base wavelet and a gap $\boldsymbol{T}$ = 16 ms. The estimated wavelet (Figure 6) is not near to zero phase and this may indicate that a phase compensation is needed (ideally to be applied deterministically through a Q compensated depth migration). To assess the goodness of estimation, we apply a phase compensation to the data that removes the phase of the estimated wavelet. The result of this process is shown in Figure 5b. One can clearly see that many reflectors (indicated by arrows) become more zero phase after this process, indicating that the estimated wavelet is reliable.

Conclusions

The use of higher order statistics for mixed phase wavelet estimation is often challenged by the consistency and the reliability of the results. Part of the reason is inherent in the nonlinearity of the problem. To improve the estimation, additional information about the wavelet can be used, such as reliable knowledge of its amplitude spectrum. This simplifies the problem to finding a phase-only wavelet that one can further constrain by its time support. The interest in bispectrum analysis and their use for wavelet estimation without phase unwrapping is revived in this abstract. The method is developed using third order statistics but extension to fourth order statistics is straightforward.

References

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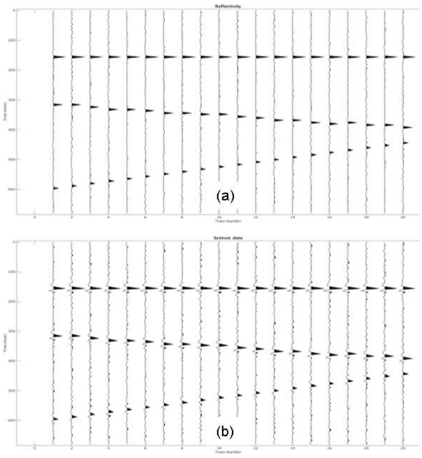

Figure 3. *synthetic reflectivity (a), corresponding synthetic data (b)*

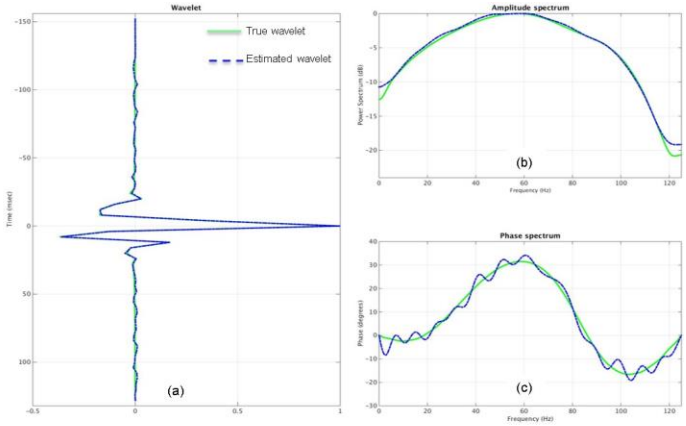

Figure 4. *Comparison between the true and the estimated wavelets (a) wavelets in time, (b) amplitude spectra and (c) phase spectra*

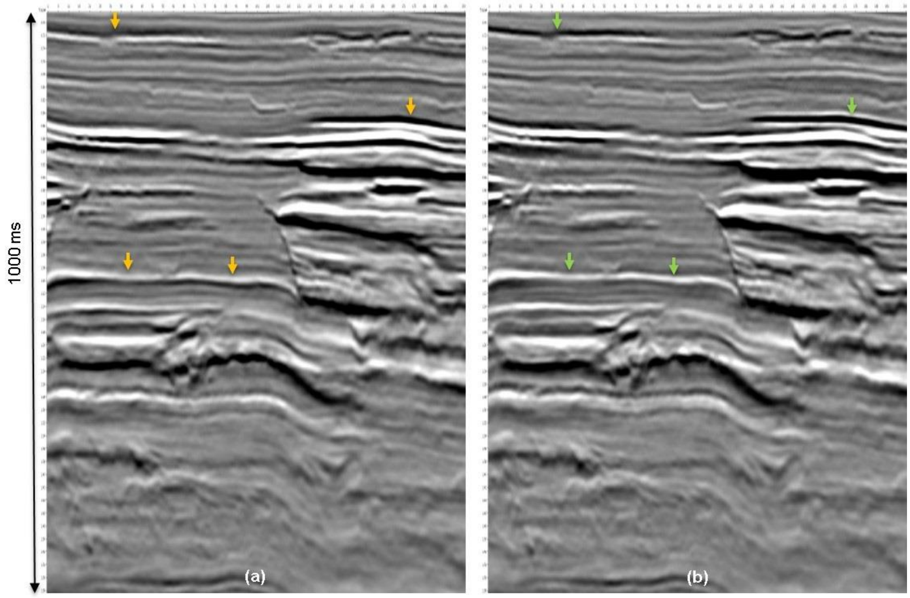

Figure 5. *Window from a PSTM subline section (a) after the application of phase compensation (b)*

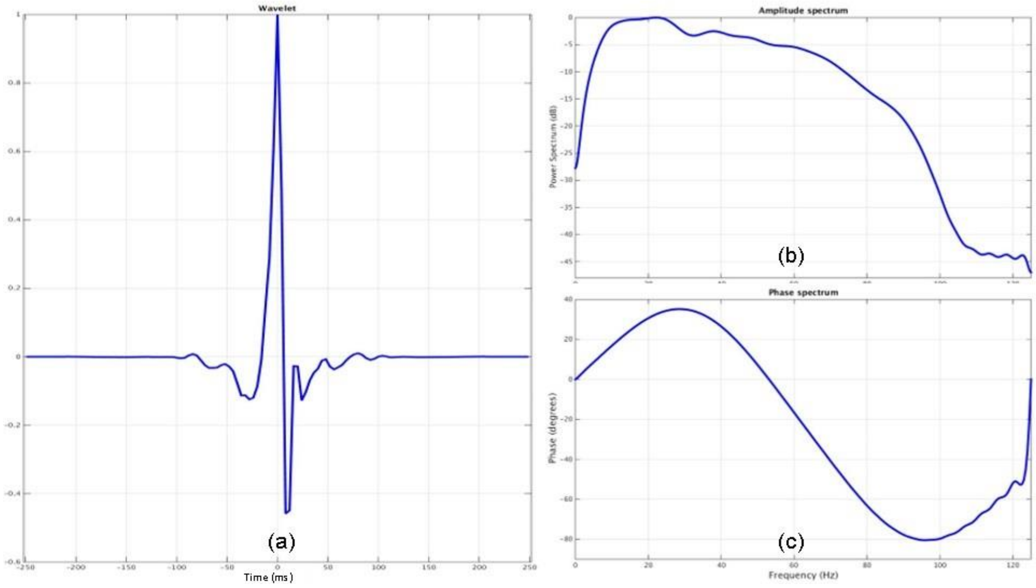

Figure 6. *Comparison between the true and the estimated wavelets (a) wavelets in time, (b) amplitude spectra and (c) phase spectra*