Full Wavefield Modeling with Vector Reflectivity

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Summary

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This work describes a method for computing the full acoustic seismic wavefield using a new two-way equation parameterized by vector reflectivity and velocity. This method is contrasted with full wavefield modeling using variable density and demonstrates the equivalence of the two methods. Thus, if an estimate of reflectivity is known or estimated the full acoustic seismic wavefield can be generated from velocity and reflectivity without explicit knowledge of density. This has an impact in any seismic inversion procedure such as Full Waveform Inversion. A modeling example is shown demonstrating the equivalence of the two methods for a known earth model. Wavefield snapshots and seismograms for both methods are shown including the cases of the following: (1) total vector reflectivity, (2) the vertical and horizontal components of reflectivity separately and (3) variable density. A second example compares recorded field data to synthetic seismograms obtained with the proposed approach, where the estimated reflectivity was extracted from a seismic image. It is noted that data misfits between the real and modeled data could be used in velocity and reflectivity inversion.

Introduction

Seismic modeling is a critical component of many applications of seismic analysis and inversion, where the modeled seismograms are compared to seismic field data. Often modeling is done with velocity only and does not include the effects of reflectivity (or density). This results in amplitude misfits when comparing the synthetic seismograms to field data. This is particularly problematic when density variations are large (e.g. reflections from the water bottom or salt-sediment interfaces). In Full Waveform Inversion (FWI) applications, the kernels typically work well for transmitted events such as refractions and turning waves. However, when targets are beyond the penetration depth of diving waves, reflections must be taken into account and modeling with only velocity usually fails. So, it important to include the effects of both velocity and reflectivity for data comparison and inversion.

This paper discusses wave-equation modeling that includes both the effects of velocity and reflectivity. While modeling can be parameterized using density, the difficulty is that the seismic response does not directly measure density. So, it is preferable to formulate the modeling equations in terms of parameters that are measured: velocities (e.g. fitting data moveout) and reflectivity derived from amplitudes. This avoids the need to estimate density from the data. This contrasts with methods where the first-order reflectivity is computed by linearizing the acoustic wave-equation by the Born approximation (e.g., Mora, 1989) with the assumption that velocity perturbation is small. A method for modeling with velocity and reflectivity exists using one-way propagation (e.g. Berkout, 1981). However, this one-way method does not include complex two-way propagation, refractions or turning waves.

The method discussed here formulates a full wave-equation in terms of vector reflectivity and velocity and thus allows for the synthesis of seismograms without explicit knowledge of density. In an inversion setting, the reflectivity is derived from seismic imaging and the velocity is derived from tomography or FWI. Examples of modeling with vector reflectivity and with variable density parametrization are compared - demonstrating the equivalence of the two. A second example compares field data to the reflectivity based modeled seismograms, where the reflectivity is derived from a seismic image. This can form a foundation for inversion, where the velocity and reflectivity are estimated iteratively.

Theory

The major purpose of this work is to provide a wave-equation based modeling method, which is formulated in terms of velocity and reflectivity. The discussion here is based on the isotropic acoustic wave-equation for pressure. It can be easily extended to anisotropic media by altering the Laplacian, but this is beyond the scope of this text. The first objective is to show the equivalence of a wave-equation using variable density to one using vector reflectivity. The wave-equation for pressure P is given by:

$$\frac{\partial^2 P}{\partial t^2} - V^2 \rho \nabla \cdot \left(\frac{1}{\rho} \nabla P\right) = S, \tag{1}$$

where V is velocity, ρ is density and S is the source. A simple change of variables eliminates the direct reference to density in this equation. This is done by replacing the density with $Z = \rho V$, where Z is acoustic impedance. It gives rise to the wave-equation in terms of acoustic impedance:

$$\frac{\partial^2 P}{\partial t^2} - V Z \nabla \cdot \left(\frac{V}{Z} \nabla P\right) = S. \tag{2}$$

To isolate the derivative of velocity from the derivative of impedance, equation (2) is expanded to the following form:

$$\frac{\partial^2 P}{\partial t^2} - \left[V^2 \nabla^2 P + V \nabla V \cdot \nabla P + V^2 Z \nabla \left(\frac{1}{Z}\right) \cdot \nabla P\right] = S. \tag{3}$$

Note that $V^2 \nabla^2 P$ is a Laplacian, which controls propagation speed, and can be modified for anisotropic wave speeds. The other terms in the brackets control amplitudes. The term $\frac{1}{2} \frac{\nabla Z}{Z}$ is the normalized rate of impedance change in each vector direction, and can be identified as a vector reflectivity:

$$\boldsymbol{R} = \frac{1}{2} \frac{\nabla Z}{Z} = -\frac{1}{2} Z \nabla \left(\frac{1}{Z}\right). \tag{4}$$

This substitution gives the final equation for wave propagation with vector reflectivity:

$$\frac{\partial^2 P}{\partial t^2} - \left\{V^2 \nabla^2 P + V \nabla V \cdot \nabla P - 2V^2 (\boldsymbol{R} \cdot \nabla P)\right\} = S. \tag{5}$$

Example 1: The equivalence of variable density and vector reflectivity modeling

This example demonstrates the equivalence of using the variable density wave-equation (1) and the vector reflectivity wave-equation (5) for modeling of the seismic response. The vector reflectivity equation is equivalent to the response due to directional changes in impedance and sums them to get the total response. In this example, the vector reflectivity was computed from a known impedance. However, in an inversion setting it is then necessary to estimate the reflectivity from the seismic image.
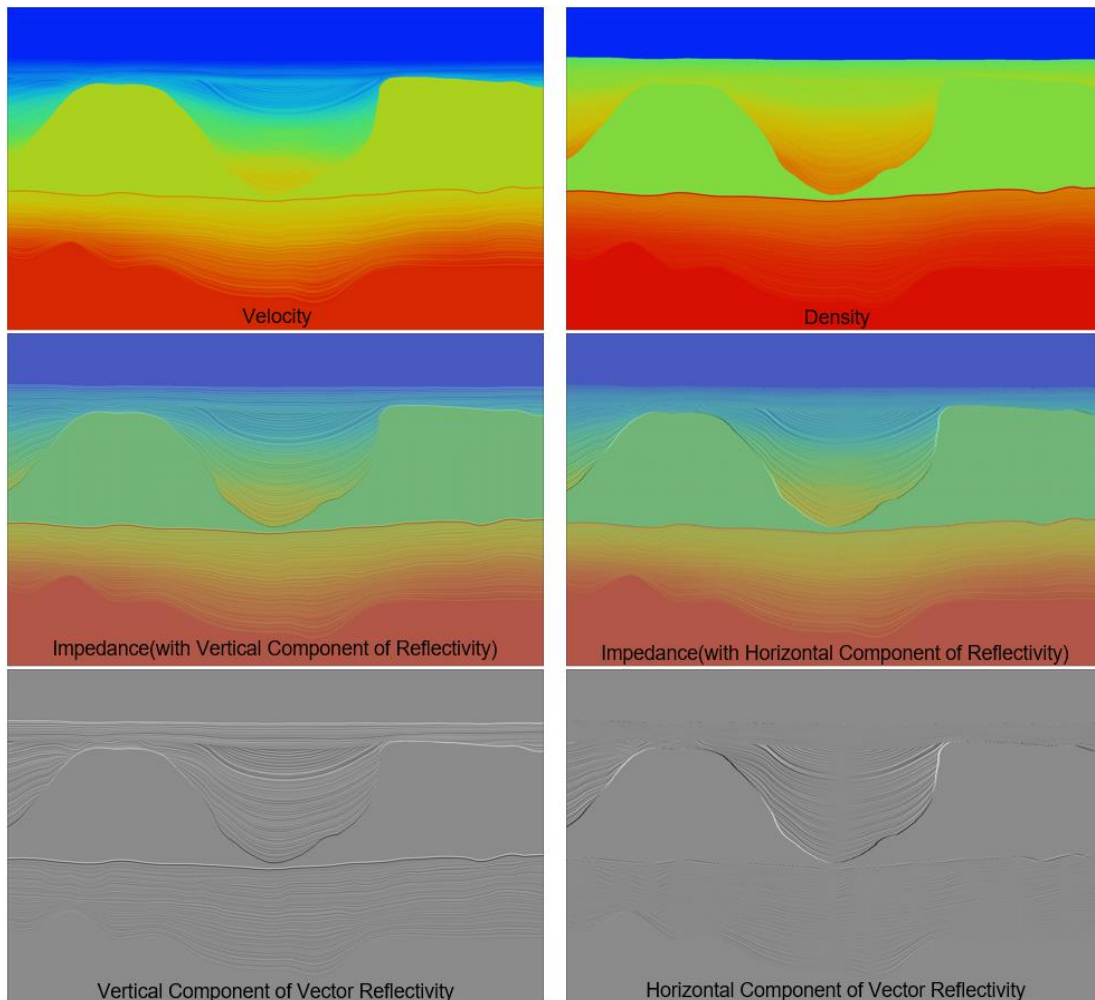
Figure 1. *Model representation of a geological setting with the presence of salt. The top row shows the original velocity and density models. The second row shows the impedance for this model (with the vertical and horizontal components of the vector reflectivity interleaved). The third row shows the vertical and horizontal components of the vector reflectivity computed from the impedance.*

Figure 2 shows the seismic response (wavefield snapshots and shot gathers) for the model displayed in Figure 1, using equations (1) and (5) for the vector reflectivity and variable density synthesis respectively. Note that the wavefield modeling using vector reflectivity creates the equivalent results as in variable density modeling, but with no need to explicitly know the density field. This is important since an accurate reflectivity model is more plausible to estimate from seismic data, in comparison with the estimation of a density field.
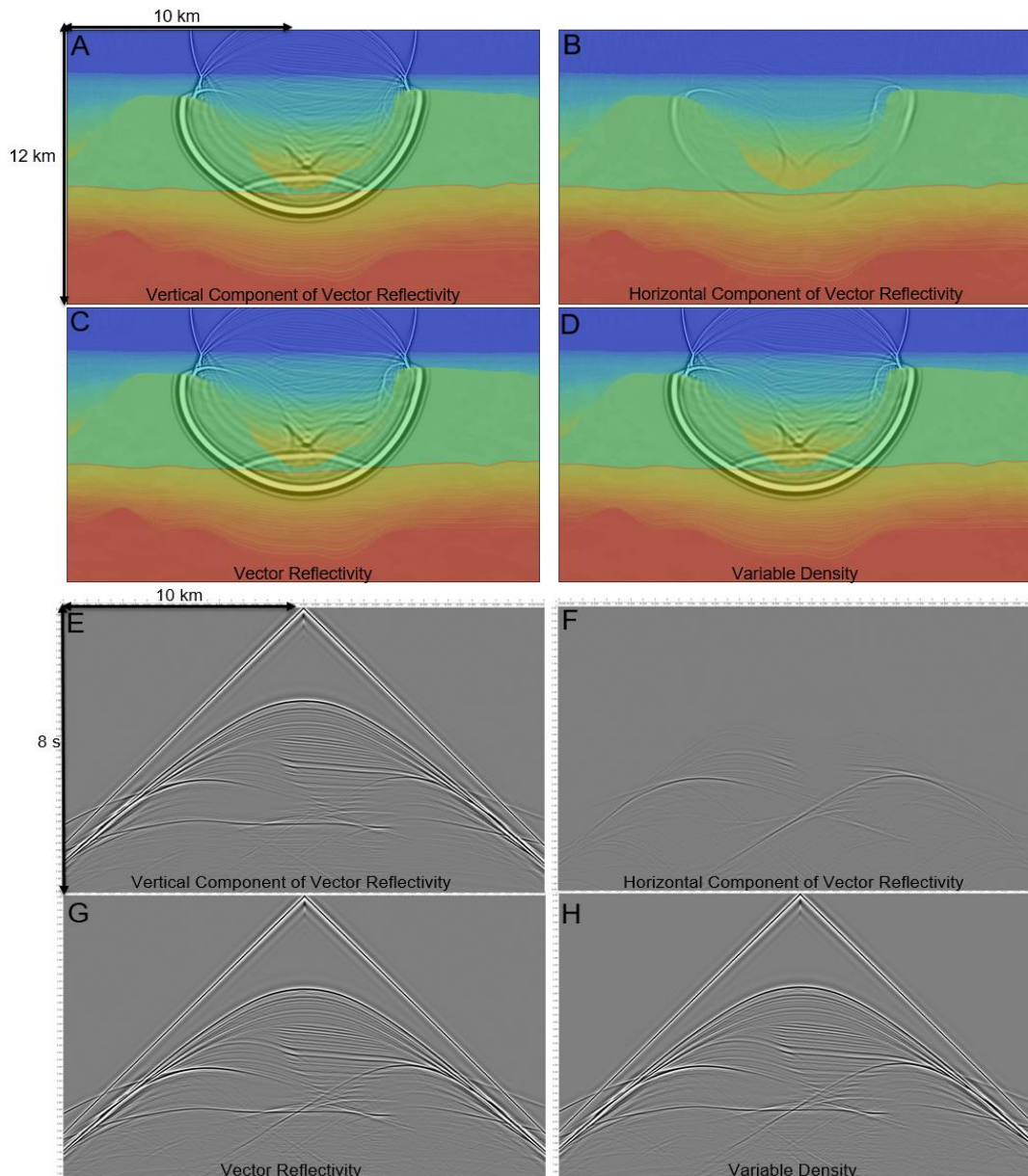
Figure 2. *Seismic response for the model described in Figure 1. Displays A and E are a wavefield snapshot and surface seismogram using the vertical component of reflectivity. Displays B and F show the results using the horizontal component of reflectivity. Displays C and G are the results for vector reflectivity. Results using variable density modeling are shown in displays D and H. Comparisons of snapshots in displays C and D and seismograms in displays G and H demonstrate the equivalence of the vector reflectivity and variable density methods. The results also show that full wavefield synthetics can be generated using reflectivity (and velocity) without explicit knowledge of density.*

Example 2: Field data example from deep water.

The previous example demonstrated that full wavefield seismograms can be generated when the reflectivity and velocity are estimated. In this example, a seismic image and a velocity field were generated for a dual-sensor survey in the Gulf of Mexico. The seismic image is a near angle stack with deconvolution imaging conditions applied and this image was used as a vertical reflectivity estimate. The reflectivity image and wavefield snapshots generated from the estimated reflectivity are shown Figure 3. The processed field data and synthetic seismograms (without and with a free surface) are shown in Figure 4. The misfit between the modeled seismograms and the real seismic data can be used to update the velocity and reflectivity in an iterative fashion. For example, FWI and least squares imaging can be used to improve velocity and reflectivity estimates.
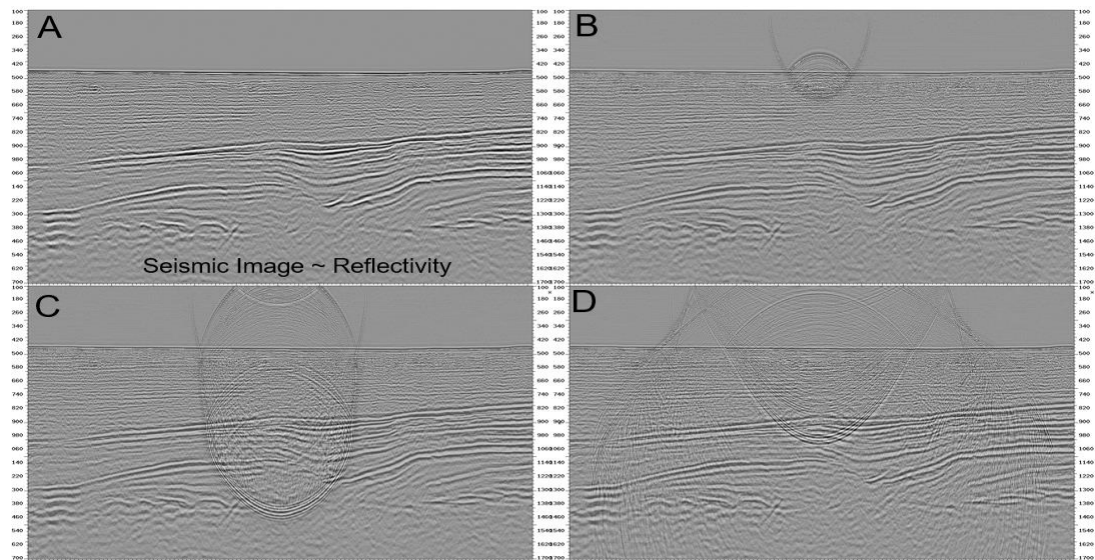
Figure 3 *A. Seismic image (used as a reflectivity estimate) overlaying the reflectivity and modeled wavefield snapshots B, C, and D from the full wavefield reflectivity modeling for three different times.*
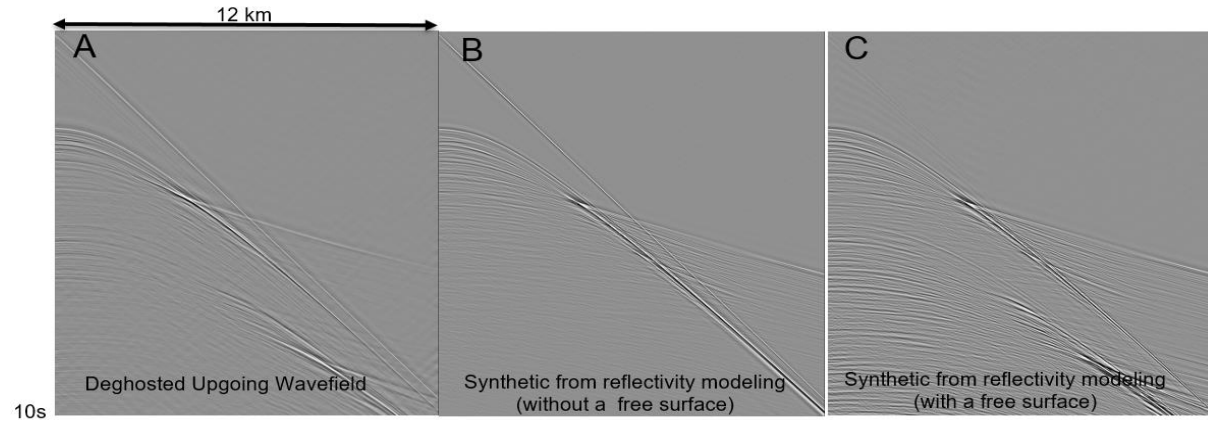
Figure 4 *A. Deghosted upgoing field data and the modeled synthetic seismograms using the full wavefield reflectivity modeling, with (B) and without (C) free surface. Note the reproduction of all the events observed in the field record, in the synthetic seismograms computed with free surface.*

Conclusions

Seismic synthetics for comparison to real field data and inversion methods should include the effects of both velocity and density (or reflectivity). The full wavefield reflectivity modeling discussed above achieves this requirement when estimates of the reflectivity and velocity are known (or estimated). The full wavefield vector reflectivity modeling produces equivalent synthetics to variable density modeling. This overcomes the problem of the availability of a density model for accurate estimation of reflections. Also, in addition to reflections, it can synthesize refractions and turning waves, which is beyond the capability of the current available methods.

Acknowledgements

References

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Mora, P. [1989] Inversion= migration+ tomography, Geophysics, **54**(12): 1575-1586.