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Least-Squares Wave Equation Migration With Angle Gathers

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Summary

Least-Squares migration (LSM) produces high-resolution images ready for reservoir characterization. It corrects the image amplitudes from the migration operator limitations and the uneven illumination created by a complex model. When posed as a reflectivity inversion in the angle domain, LSM compensates the angle gathers amplitudes. Our LSM algorithm explicitly computes a Hessian matrix or point spread functions (PSF) with an extra angle dimension. It applies a chain of operators and their adjoints for modeling, migration, and angle generation to a grid of point scatterers distributed through the model. The angular reflectivity is recovered by solving a linear system of equations that deconvolves the multidimensional PSF from the migrated image gathers. The implementation is efficient and effectively incorporates the spatial variability of the PSF. Results from Sigsbee2A model and a multisensor streamer survey from the Central North Sea show how our procedure improves the image resolution and the AVA reliability.

Introduction

Seismic image amplitudes are biased by the earth model, acquisition parameters, and imaging operators. The bias is particularly prominent in the presence of complex earth models, as it can affect the interpretation of amplitudes and their variability with angle. Provided with a good velocity model, migration generates flat angle gathers with smooth variations in reflectivity. Abrupt changes in amplitude with angle are typically indicative of illumination problems.

Seismic imaging operators (i.e. modeling/migration) are non-unitary (Claerbout, 1992), meaning that if **L** is a modeling operator, and **L′** is its adjoint (migration), their product **H=L'L** is not the identity matrix; **H** is a Hessian matrix, whose elements are the point spread functions (PSF). As a result, depth migrated images are blurred, and AVA is often not preserved. This can change depending on the migration operators as wave equation one-way (WEM), two-way (RTM) or asymptotic (Kirchhoff) handle the amplitudes differently and have different degrees of kinematic accuracy (Gray et al., 2001, Zhang et al., 2005). Asymptotic Kirchhoff operators are closer to be unitary (Bleistein, 1987) and are usually trusted for AVA interpretation in spite of their kinematic limitations in high-contrast rapidly varying earth models. As our ability to estimate more detailed earth models increases by the usage of Full Waveform Inversion (FWI) it is desirable to utilise more accurate wave equation operators that could adequately preserve image amplitudes.

Here, we discuss a least squares migration (LSM) solution that balances the depth migrated images to account for uneven illumination, reduces the image blurring, and corrects the angle gather amplitudes. The algorithm assumes that the background earth model is accurate and poses the estimation of the reflectivity as a least-squares inversion problem in the reflection angle domain. It explicitly computes the Hessian matrix with an angle dimension by applying a chain of operators (modeling/migration and offset to angle transforms) to a grid of point scatterers distributed throughout the model space. The method assumes a degree of stationarity of the PSF as they are later interpolated to fully populate the image space. As a final step, it solves a linear system where the migrated images and the PSFs are the known quantities, and the angular reflectivity is unknown. Results from the Sigsbee2A model and a multisensor field survey from the central North Sea demonstrate the advantages of the method.

Least-Squares Migration with gathers

The Least-squares migration algorithm can be summarized as follows: given a linear modeling operator **L** and a reflectivity model **m**, compute synthetic data **d** using the relation **d** = **Lm**, form a quadratic cost function

$$S(\mathbf{m}) = \left\| \mathbf{d} - \mathbf{d}_{obs} \right\| = \left\| \mathbf{Lm} - \mathbf{d}_{obs} \right\|, \qquad (1)$$

and seek a reflectivity model that minimizes it.

A closed form solution for the least-squares estimate of **m** is given by:

$$\hat{\mathbf{m}} = \left(\mathbf{L}'\mathbf{L}\right)^{-1}\mathbf{L}'\mathbf{d}_{obs} \qquad (2)$$

$$\hat{\mathbf{m}} = \mathbf{H}^{-1}\mathbf{m}_{mig}, \qquad (3)$$

where the migration operator **L′** is the adjoint of the modeling operator **L**, $\mathbf{m}_{mig}$ is the migrated image, and **H** is the Hessian matrix whose elements are the point spread functions (PSF). Equation 3 implies that the reflectivity can be estimated by a matrix-vector multiplication of the inverse of the Hessian ($\mathbf{H}^{-1}$) times the migrated image ($\mathbf{m}_{mig}$). However, it is not numerically feasible to compute the inverse Hessian matrix for most field data applications. Alternately, a low-rank approximation to the inverse of the Hessian has been proposed in the literature (e.g. Guitton, 2004).

A better approach is to explicitly compute the Hessian matrix and estimate the reflectivity (Valenciano, 2008) rather than approximating the matrix inverse. This alternative solution is obtained by solving the linear system:

$$\mathbf{H}\hat{\mathbf{m}} = \mathbf{m}_{mig}, \qquad (4)$$

using an iterative inversion algorithm (e.g. conjugate gradients). To generalize equation 4, and invert for angular reflectivity, we need to define the Hessian in the prestack image space.

Expanding Hessian dimensionality to the angle domain

Valenciano and Biondi (2006) defined the Hessian matrix in the prestack image domain as a chain of operators from the subsurface offset $\mathbf{h} = (h_x, h_y)$ to the reflection and azimuth angle $\Theta = (\theta, \alpha)$:

$$\mathbf{H}(\mathbf{x},\Theta,\mathbf{x}',\Theta') = \mathbf{T}'(\Theta,\mathbf{h})\mathbf{H}(\mathbf{x},\mathbf{h},\mathbf{x}',\mathbf{h}')\mathbf{T}(\Theta',\mathbf{h}'), \qquad (5)$$

where the operator $\mathbf{T}$ defines the transformation from reflection and azimuth angle to subsurface offset (Sava and Fomel, 2003). The Valenciano and Biondi (2006) approach can be applied to any prestack volume where angle gathers may be produced from direct binning using Poynting Vectors (Yoon and Marfurt, 2006) or using extended imaging conditions (Sava and Fomel, 2005). After computing the angle domain Hessian, the linear system from equation 4 can be expanded to estimate the least-squares angular reflectivity (Valenciano 2008):

$$\mathbf{H}(\mathbf{x},\Theta,\mathbf{x}',\Theta')\hat{\mathbf{m}}(\mathbf{x},\Theta) = \mathbf{m}_{mig}(\mathbf{x},\Theta). \qquad (6)$$

Computing the angle domain Hessian matrix

Here, we compute the Hessian matrix in the angle domain by applying the chain of operators from equation 5 to a grid of point scatterers distributed throughout the model space. The spacing of the point scatters is controlled by several factors including acquisition geometry, medium velocity, and imaging frequency. Here we assume local stationarity of the PSF as they are later interpolated to have a contribution at each image point.

The Sigsbee model

The Sigsbee2A model (Figure 1a) is ideal for illustrating the variable illumination on the angle gathers. We generated synthetic data with constant amplitude angle gathers (i.e. no AVA). As expected, however, the migration (WEM) angle gathers (Figure 1b) show uneven illumination—noticeably under the salt. In contrast, the LSM angle gathers (Figure 1c) show the expected AVA in the sediments, and lesser variability than migration below the salt. Figure 1d shows a comparison of the amplitudes extracted in the red and green circles from figures 1b and 1c. The LSM produces constant amplitude for all angles, as expected.

Field data from the North Sea

A 3D narrow-azimuth multisensor streamer dataset from the Central North Sea (Viking Graben) further illustrates the advantages of LSM. Here, the presence of a complex overburden (high-velocity bodies: “V bright”) produces uneven illumination at the reservoir level. Figure 2 shows a comparison of the results from migration (WEM) and LSM. The LSM improves resolution (Figure 2) and enables discrimination of the reservoir from the background. Figures 2e and 2f show angle gathers at the target and their corresponding AVA. The illumination compensation with LSM changes the AVA trend as well as the interpretation at the reservoir (Figure 2g). The LSM AVA trend matches the response predicted by AVA modeling from a nearby well.

Conclusions

We presented a wave equation LSM solution that produces reliable AVA in complex media. Synthetic and field data examples show improvement after LSM in image resolution and AVA consistency. We showed on the field data from the Central North Sea (Viking Graben) that the LSM illumination compensation can change the AVA interpretation at the reservoir level. We conclude that LSM can be a robust solution for producing volumes of angular reflectivity ready for reservoir characterization.

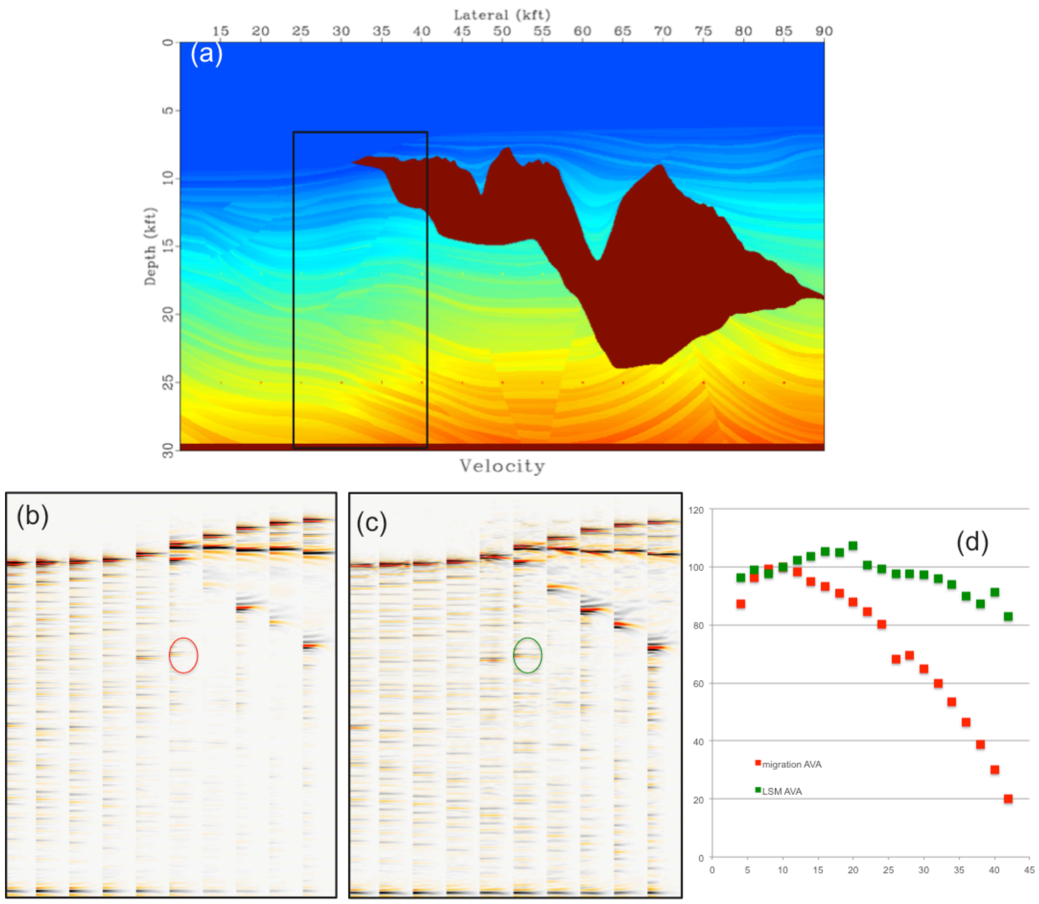

Figure 1 *Sigsbee2A: (a) velocity model, (b) Migration (WEM) angle gathers, (c) LSM angle gathers, and (d) AVA comparison at the reflector in the center of the circle. The LSM compensates for uneven illumination underneath the salt body.*

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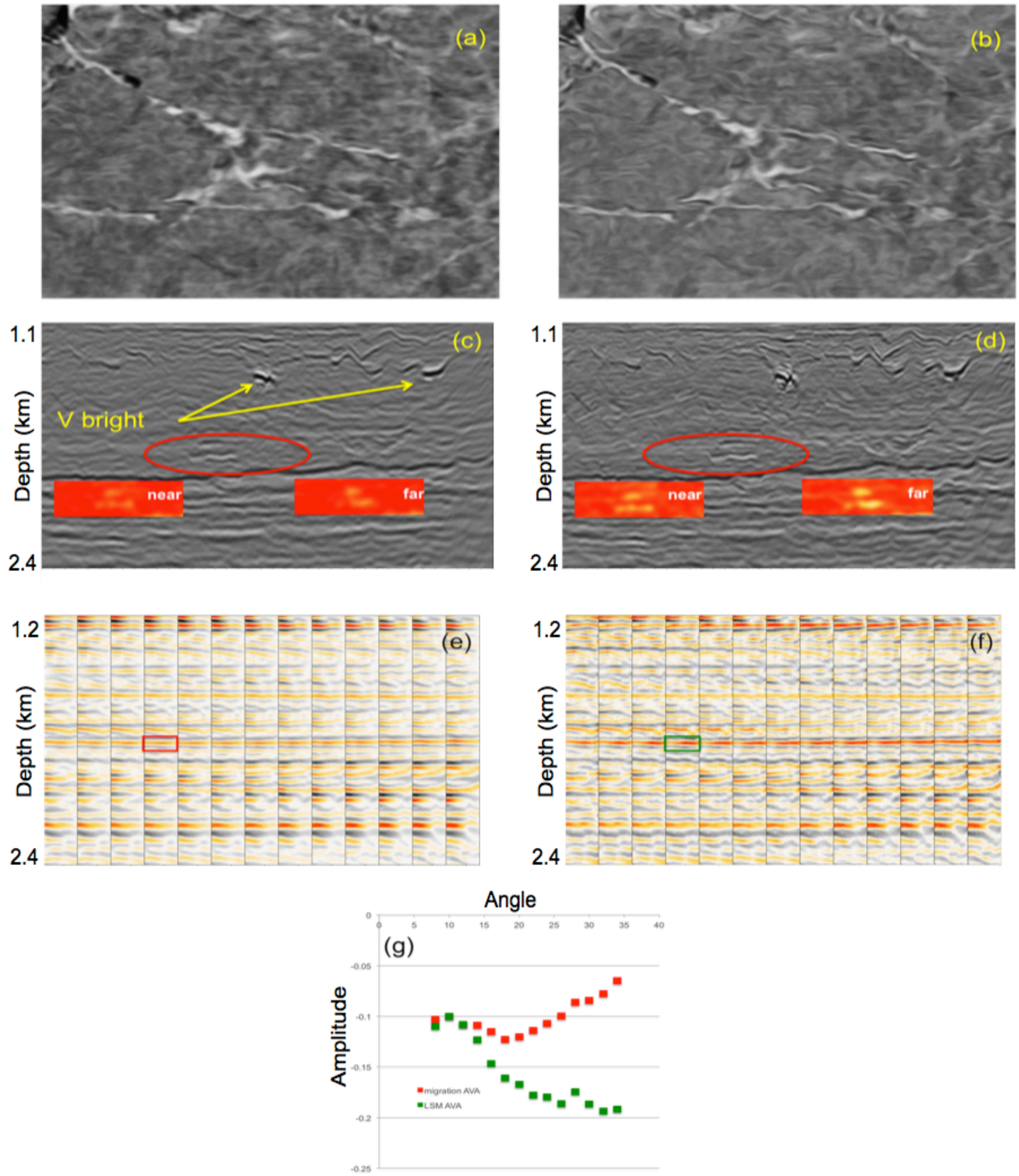

Figure 2 *Central North Sea data: (a) Migration (WEM) depth slice, (b) LSM depth slice, (c) Migration inline section, (d) LSM inline section, (e) Migration angle gathers, (f) LSM angle gathers, and (g) AVA comparison at the reservoir depth. The rectangles in color in panels (c) and (d) show the RMS amplitude values inside the red ellipses. Note that the near vs far angle stacks can better discriminate the reservoir from the background in the LSM image.*