

# Subsalt Imaging by Wave Equation Reflectivity Inversion

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## SUMMARY

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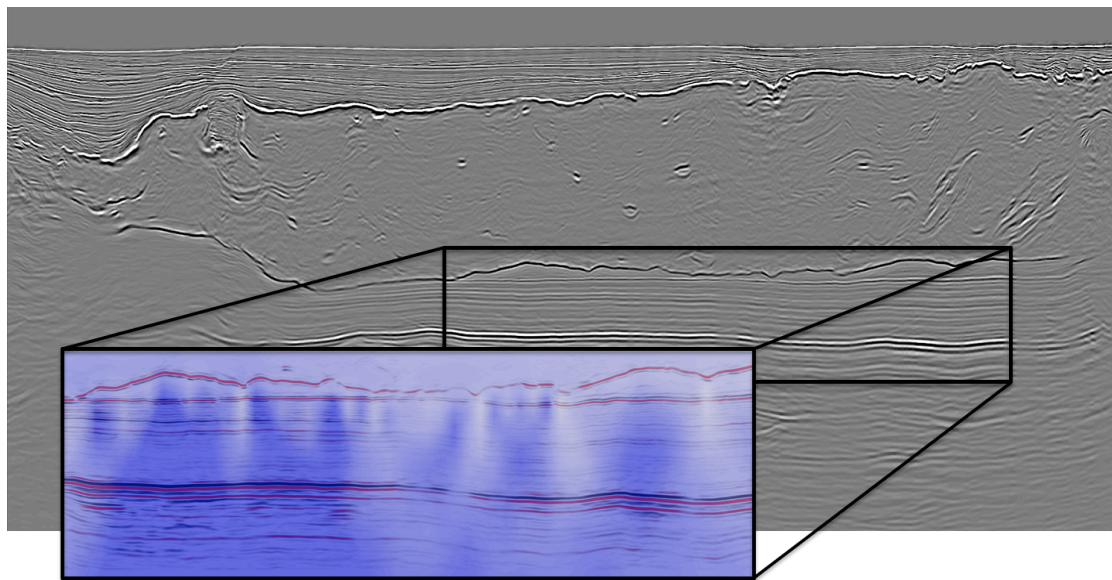
Complex propagation of seismic waves through salt bodies, combined with incomplete acquisition geometry, result in uneven illumination of subsalt targets. Consequently, amplitude information from migrated images can be distorted. We propose the use of a wave equation reflectivity inversion (WEI) to mitigate non-geological amplitude variations. WEI requires explicit computation of the point-spread functions (PSFs), as part of a linear inversion workflow. The implementation is efficient, and successfully reduces the effects of irregular illumination. Results from synthetic and field data experiments demonstrate how WEI mitigates amplitude distortions and enhances the resolution of migrated images. The WEI enhanced images are readily available for attribute analysis and reservoir studies.

## Introduction

Incomplete acquisition geometries and complex overburden cause illumination variations distorting the seismic amplitudes on the migrated image. To alleviate this problem, innovative acquisition configurations such as Multi-azimuth (MAZ), Wide-azimuth (WAZ), or Full-azimuth (FAZ) have been proposed. However non-uniform illumination remains a challenge particularly below salt bodies and other complex geological structures. It leads to uncertainty in estimation of rock properties from the migrated volumes. Figure 1 displays an image from a FAZ dataset acquired in the Gulf of Mexico (GoM). The illumination map (overlaid on the image) demonstrates the distortions of the seismic energy propagated through a rugose base salt. The variable illumination creates significant amplitude variations that are unrelated to the lithology.

To mitigate the irregular illumination problem in the subsalt area, we propose a wave equation reflectivity inversion (WEI) method. The method poses depth imaging as least squares problem (Nemeth et al., 1999, Prucha and Biondi, 2002) with explicit computation of the Hessian or point-spread function (PSF) (Valenciano et al., 2006). First, we estimate the Hessian in the target area, and then solve a linear system of equations to deconvolve the multidimensional PSFs from the migrated image. WEI not only removes the illumination imprint on the seismic amplitudes, it also enhances the wave number content, improving the overall resolution of the migrated image.

We apply the proposed wave equation reflectivity inversion method to the Sigsbee2A synthetic dataset. WEI recovers the amplitudes in the subsalt shadow zone and improves the sharpness of the reflectors. Application of WEI to a GoM field dataset alleviates illumination artifacts and enhances spatial resolution.



*Figure 1* Depth image of the Gulf of Mexico FAZ dataset. Zoomed area focuses on the base and subsalt region, showing the seismic reflectivity overlaid with the illumination map. Dark blue colour represents high illumination while light blue colour corresponds to low illumination.

## Theory

Seismic data  $\mathbf{d}$  can be described as the result of a linear modelling operator  $\mathbf{L}$ , applied to a reflectivity model  $\mathbf{m}$ :

$$\mathbf{d} = \mathbf{L}\mathbf{m} \quad (1)$$

A least-squares approach can be used to estimate the reflectivity model  $\mathbf{m}_{\text{est}}$  from the observed data  $\mathbf{d}_{\text{obs}}$  by minimizing the objective function:

$$\mathbf{J} = \|\mathbf{d} - \mathbf{d}_{\text{obs}}\|^2 = \|\mathbf{L}\mathbf{m} - \mathbf{d}_{\text{obs}}\|^2 \quad (2)$$

The reflectivity  $\mathbf{m}_{\text{est}}$  that minimizes the objective function becomes:

$$\mathbf{m}_{\text{est}} = (\mathbf{L}'\mathbf{L})^{-1}\mathbf{L}'\mathbf{d}_{\text{obs}} = \mathbf{H}^{-1}\mathbf{m}_{\text{mig}} \quad (3)$$

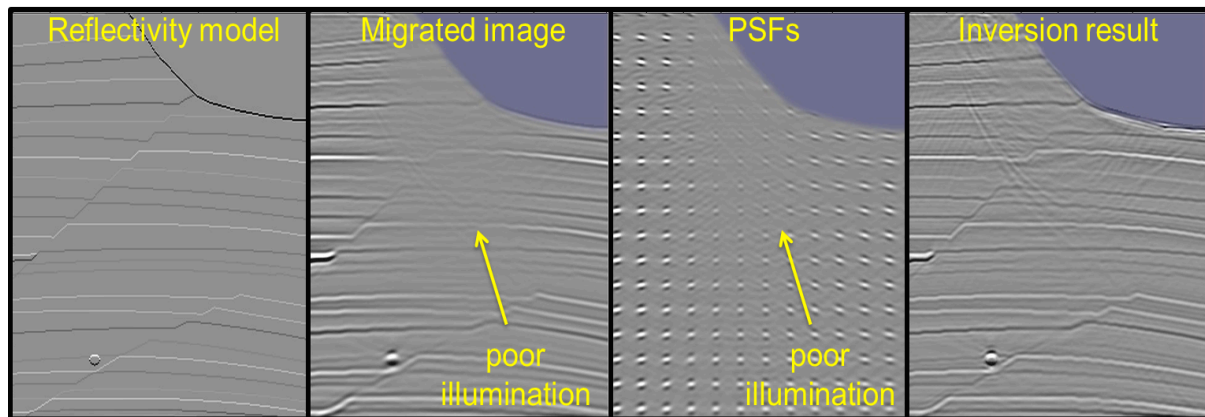
where  $\mathbf{L}'$  is the migration operator or the adjoint of the modelling operator  $\mathbf{L}$ ;  $\mathbf{m}_{\text{mig}} = \mathbf{L}'\mathbf{d}_{\text{obs}}$  is the migrated image, and  $\mathbf{H} = \mathbf{L}'\mathbf{L}$  is the Hessian matrix. The Hessian can be interpreted as the response of a point scatterer through the imaging system, or the point spread function (PSF). It contains information about acquisition geometry and earth properties. The diagonal of the Hessian matrix is a measure of the illumination. It is possible to compensate for some of the blurring and distortion effects on the image once the PSFs are known (Gelius et al., 2002, Lecomte, 2008).

We estimate the PSFs by modelling the response of a set of point scatterers distributed over the earth model, then migrating the resulting data. The computed PSFs are interpolated to the imaging grid before the inversion. Finally, we iteratively solve a linear system of equations to deconvolve the multidimensional PSFs from the image.

### Synthetic data example – Sigsbee2A model

We use the Sigsbee2A synthetic dataset to demonstrate how WEI mitigates illumination problems in a subsalt area. Figure 2, panel 1 shows the reflectivity model in the target section. The salt body creates a shadow zone on the migrated image (Figure 2; panel 2). We compute the PSFs in the subsalt zone (Figure 2; panel 3) and use them in WEI to improve the migrated result. As expected, the PSFs capture the low illumination pattern. The inversion balances the amplitudes in the target area and improves the resolution of the image (Figure 2; panel 4).

The WEI result contains some events that are not present in the reflectivity model. These events are noise coming from internal multiples being imaged in the wrong place (Valenciano, 2008). Ideally, noise needs to be removed before the inversion.

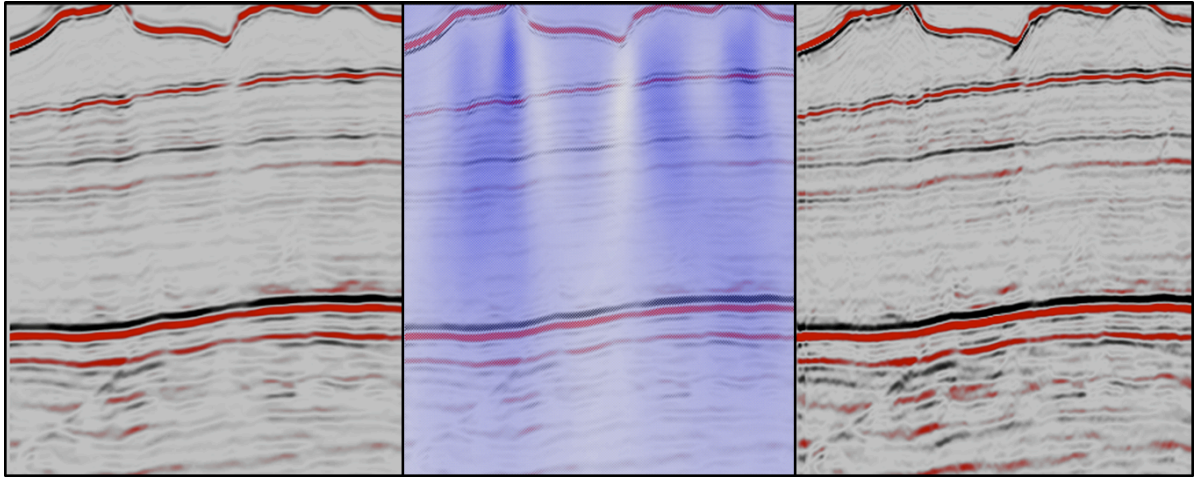


**Figure 2** Subset of the Sigsbee synthetic dataset. From left to right: reflectivity model (panel 1), migrated image (panel 2), computed point spread functions (panel 3) and WEI result (panel 4). Blue colour highlights the salt body.

### Field data example – FAZ from Gulf of Mexico

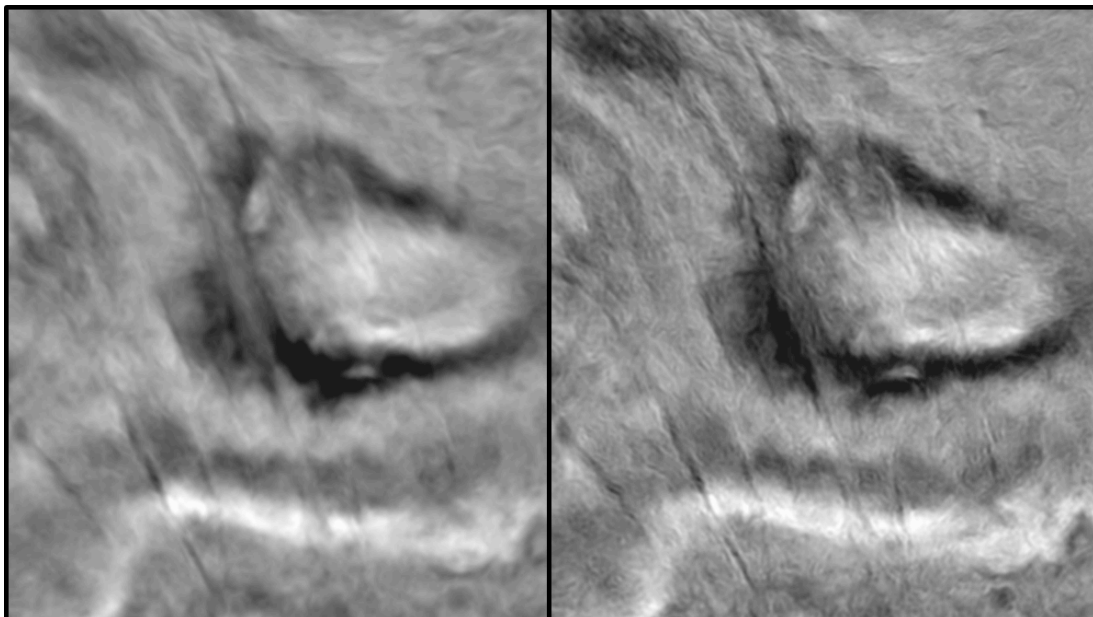
We applied the wave equation reflectivity inversion workflow to a subset (340 km<sup>2</sup>) of a FAZ dataset from the Gulf of Mexico (Long et al., 2014). Figure 1 displays the illumination map computed from the PSFs in a subsalt area. The illumination changes rapidly along the reflectors due to the focusing and defocusing of the seismic energy produced by the rugose base of salt. Figure 3 illustrates the

application of WEI to the migrated image. The illumination patterns correlate with the amplitude variations on the migrated section. WEI alleviates the effects of the irregular illumination, improving the continuity of the target reflectors.



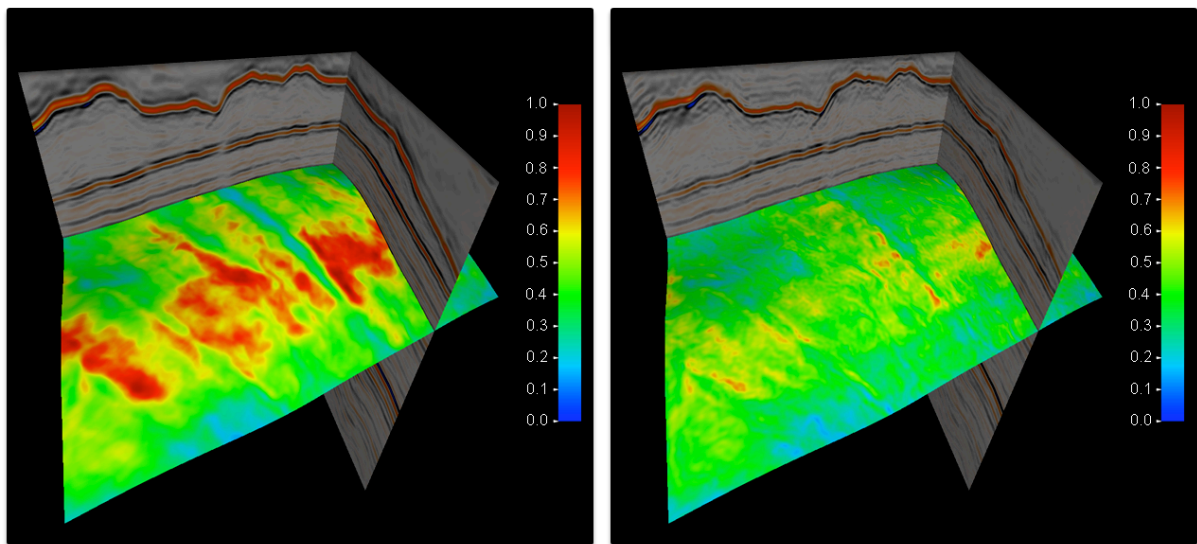
**Figure 3** Subset of the FAZ Gulf of Mexico dataset: migration result (left), illumination map overlaid on the migrated image (middle) and WEI result (right). Dark blue colour in the middle panel represents high illumination while light blue colour corresponds to low illumination.

Figure 4 displays a depth slice before and after inversion to demonstrate the improvement in spatial resolution after WEI. The image after inversion is more focused; also note the sharpness of the faulting system.



**Figure 4** Depth slice in the subsalt area before (left) and after (right) WEI. Note the improvement in spatial resolution and the sharpness of the faults.

Figure 5 displays the RMS amplitudes extracted along a key horizon before and after WEI. The inversion removes most of the irregular illumination effects, making the amplitudes more reliable for quantitative interpretation.



**Figure 5** Amplitudes extracted along target horizon before (left) and after (right) WEI.

## Conclusions

The point spread functions capture illumination variations beneath the salt bodies. This information is used in the wave equation reflectivity inversion workflow to alleviate the effects of irregular illumination. Results from synthetics demonstrate the ability of WEI to recover the seismic amplitudes in subsalt shadow zones and enhance the resolution of the image. A field data example shows improvements of the subsalt reflectivity and spatial resolution after inversion.

## Acknowledgements

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