Visco-acoustic Reverse-time Migration Using the Pseudo-analytical Method

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SUMMARY

We derive a new reverse-time migration solution that uses a pseudo-analytical, two-way extrapolator in order to compensate for attenuation losses in visco-acoustic media. The algorithm permits large time stepping and shares the advantages of spectral methods in allowing coarse computational grids. The extrapolation accounts for spatial variations in attenuation by using a series expansion approximation of the equations for constant attenuation. We apply the new scheme to a dual-sensor field survey from the North Sea, in which the resolution of the images is clearly improved after compensating for attenuation.

Introduction

Seismic waves attenuate as they travel through the Earth. This results in a biased loss of high-frequency energy, as well as a frequency-dependent variation in phase (dispersion). Therefore, to accurately perform imaging in realistic media, the amplitude loss and the phase distortion should be compensated in the migration operators. This class of operators solves the visco-acoustic wave equation, and requires an extra parameter (the quality factor *Q*) in order to accurately model the waves propagating in an attenuating media.

Two main wave equation migration methods can be used to compensate for visco-acoustic effects in depth migration. The first method uses one-way, wave-equation operators to perform depth extrapolation in the frequency domain. The frequency domain enables attenuation by turning the phase velocity into a complex quantity (Valenciano *et al.*, 2011). The second method solves the visco-acoustic, two-way wave equation in the time domain (*e.g.*, Deng and McMechan, 2008). This solution is based on a superposition of mechanical elements that are approximations to the constant-*Q* behavior defined within the frequency band of the data (*e.g.*, Emmerich and Korn, 1987). The more elements used in the superposition, the more accurate the approximation, at the expenses of more computation. On the other hand, a more accurate time- domain approach based on the exact, constant-*Q* behavior (*e.g.*, Kjartansson, 1979) requires the use of fractional time derivatives. The accurate computation of the fractional derivatives requires the use of large memory resources.

To overcome this problem, Chen and Holm (2004) introduced the first algorithm to simulate attenuation from fractional wave equations by using a fractional Laplacian that can be computed in the Fourier domain. The initial approximations using this concept (*e.g.*, Carcione, 2010) did not split the effects of dispersion and absorption into different operators - an algorithmic feature that can be convenient for a practical migration implementation. Recently, Zhu and Harris (2014) derived a wave equation which splits those terms, and which Zhu *et al.* (2014) solved by the pseudo-spectral approach within a reverse time migration algorithm.

Here, we introduce attenuation compensation in RTM by using a pseudo-analytical extrapolation (Etgen and Brandsberg-Dahl, 2009). Our method is based on the two-way wave equation derived by Zhu *et al.* (2014), with the introduction of an approximation to the dispersive term of the equation that accurately simulates the visco-acoustic propagation in heterogeneous media. As a consequence, we obtained an efficient algorithm that allows the use of coarse spatial grids and large time steps without jeopardizing the accuracy of the wavefield propagation. We show an example of the successful application of this new scheme using field data from the North Sea in which the image resolution is clearly improved after compensating for attenuation.

Two-wave wave equation in a visco-acoustic media and Q-RTM implementation

Assuming the constant-*Q* dispersion relation introduced by Kjartansson (1979), Zhou and Harris (2014) derived the two-way wave equation for visco-acoustic, isotropic, heterogeneous media with constant density as:

$$\frac{1}{c_0^2(\mathbf{x})}\frac{\partial^2\sigma(\mathbf{x},t)}{\partial t^2} = \eta(\mathbf{x})\left(-\nabla^2\right)^{\gamma+1}\sigma(\mathbf{x},t) + \tau(\mathbf{x})\frac{\partial}{\partial t}\left(-\nabla^2\right)^{\gamma+1/2}\sigma(\mathbf{x},t), \qquad (1)$$

where the coefficients as a function of space are defined as $\eta(\mathbf{x}) = -c_0^{2\gamma(\mathbf{x})}\omega_0^{-2\gamma(\mathbf{x})}\cos(\pi\gamma(\mathbf{x}))$, $\tau(\mathbf{x}) = -c_0^{2\gamma(\mathbf{x})-1}\omega_0^{-2\gamma(\mathbf{x})}\sin(\pi\gamma(\mathbf{x}))$, where $\gamma = \pi^{-1}\tan^{-1}(1/Q)$. Q is the quality factor, and c_0 and ω_0 are the reference velocity and frequency, respectively. The advantage of eq. (1) for migration is that in the right-hand term, the dispersion (first) and attenuation (second) operators are split. However, a drawback is found in the dispersive operator computed in the wavenumber domain for an attenuating media with spatially variable Q. In this case, there is an ambiguity regarding the value of γ in the exponent that should be used to compute the operator in the wavenumber domain. Zhu and Harris (2014) proposed that γ should be computed from an average of the different Q values of the model. However, the selection of a unique γ compromises the accuracy of the arrival times of the different events in a Q-variable media.

We overcome this problem with a series expansion of the operator for dispersion that allows a better approximation for propagation in heterogeneous media. We begin with the frequency (ω)–wavenumber (k) version of eq. (1)

$$\frac{\omega^2}{c^2} \approx \eta\left(k^2\right)^{\gamma+1} + (i\omega)\tau\left(k^2\right)^{\gamma+1/2} \tag{2}$$

Assuming that a function of any variable (γ in this case), can be approximated around a certain value of $\gamma=\gamma_0$ as

$$L(\gamma) \approx L(\gamma_0) + \Delta L(\gamma), \tag{3}$$

it is easy to prove that the first term in the right-side of eq. (2) can be approximated around $\gamma=0$ as

$$\eta\left(k^2\right)^{\gamma+1} \approx \eta\left(k^2\right) + \eta\gamma\left(k^2\right)\left[\log\left(k^2\right)\right]. \tag{4}$$

Thus, the variability of the pseudo-Laplacian operator k^2 as a function of γ is eliminated. Instead, it is applied in the space domain.

Crawley *et al*. (2010) showed that the pseudo-analytic method allows the use of coarse time and space discretization without compromising the accuracy of the modeled wavefield, even after long propagations in time and space. Since the first-order time-derivative in the second term of eq. (1) affects only the amplitude of the wavefield, the pseudo-analytic form of the Laplacian operator is still valid to correct for the error incurred when the second-order time derivative is approximated with a second-order finite-difference. The new equation in its time-discretized, pseudo-analytic form is

$$\begin{aligned}\sigma^{n+1} &= 2\sigma^{n} - \sigma^{n-1} + c_o^2(\mathbf{x})\eta(\mathbf{x})FT^{-1}\left\{\tilde{f}(\mathbf{k})\sigma^{n}(\mathbf{k},t)\right\} + \\ &c_o^2(\mathbf{x})\eta(\mathbf{x})\gamma(\mathbf{x})FT^{-1}\left\{\tilde{f}(\mathbf{k})\log\left(\tilde{f}(\mathbf{k})\right)\sigma^{n}(\mathbf{k},t)\right\} + \\ &c_o^2(\mathbf{x})\tau(\mathbf{x})\left[FT^{-1}\left\{\left(\tilde{f}(\mathbf{k})\right)^{\gamma+1/2}\sigma^{n}(\mathbf{k},t)\right\} - FT^{-1}\left\{\left(\tilde{f}(\mathbf{k})\right)^{\gamma+1/2}\sigma^{n-1}(\mathbf{k},t)\right\}\right]\end{aligned} \tag{5}$$

where FT^{-1} stands for the inverse Fourier Transform and the normalized pseudo-analytic Laplacian (Chiu and Stoffa, 2011) is defined as

$$\tilde{f}(\mathbf{k}) = \frac{2\cos\left(c_0\Delta t|\mathbf{k}|\right) - 2}{-c_0^2\Delta t^2|\mathbf{k}|^2} \tag{6}$$

For the QRTM implementation, we follow the work of Zhu *et al*. (2014), which shows a detailed discussion of the main steps to consider for the correct compensation for Q. One of the advantages of visco-acoustic wave equations in the form shown in Equations (1) and (5) is that the same algorithm can be used during the forward modeling and RTM extrapolation, with only a change of sign in the amplitude-loss term.

Visco-acoustic propagation

Figure 1 shows four panels (z,x) split by the dark lines corresponding to a wave front propagated through four different types of media: (a) acoustic, (b) amplitude-loss only, (c) dispersive only, and (d) visco-acoustic. The reference frequency is higher than the frequency content of the source wavelet used in the simulations. As a consequence, the propagated wavefields in the dispersive media (Figures. 1c and 1d) travel with a slower velocity than the background velocity; *i.e*., they have traveled a shorter distance with respect to the acoustic and amplitude-loss only media, for the same travel time.

Q-RTM field data example

We applied the Q-RTM to a 3D field dataset that was acquired with dual-sensor technology, in the North Sea. The surveyed area is characterized by gas clouds that limit the resolution of the images at the depth of the reservoir. Valenciano and Chemingui (2013) derived a Q tomographic model, and used it for attenuation compensation in a one-way wave-equation migration (WEM) (Valenciano *et al.*, 2011). Figure 2 shows the performance of our RTM algorithm, with and without Q compensation, for a maximum frequency of 50 Hz. As shown, Q-compensation greatly improves the continuity and resolution at the reservoir level. This is confirmed by observing the amplitude spectra of the images after depth to time conversion in Figure 3. It shows that the algorithm recovers the high-frequency content of the image, when compared to that image without Q-compensation.

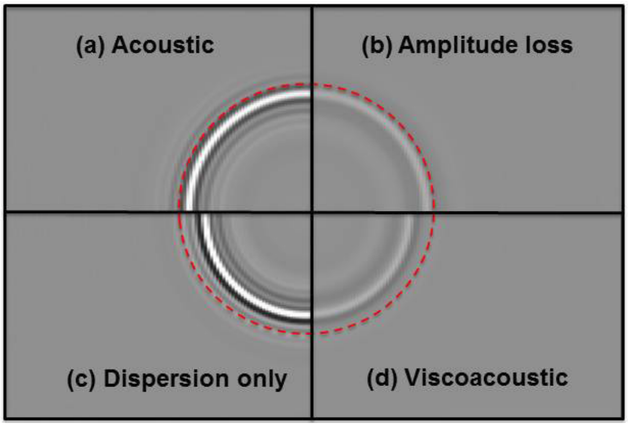

Figure 1. Snapshots corresponding to the same travel time, for wavefronts traveling through different media: (a) Acoustic, (b) amplitude-loss only, (c) dispersive only and (d) visco-acoustic. Red dash line corresponds to the reference wavefront in the acoustic case.

Conclusions

We present a reverse-time, migration algorithm that uses a pseudo-analytical extrapolator in order to solve a visco-acoustic, two-way, wave equation. Our extrapolator includes a new approximation of the dispersive term of the visco-acoustic operator. It allows for a better approximation of the waves propagation in heterogeneous attenuating media. The numerical solution enables efficient computation without compromising the accuracy of the simulated wavefields, even for long propagations in time and space. Significant improvements in image quality can be achieved by incorporating attenuation in the migration. This is demonstrated by the migration of a dual-sensor, towed-streamer survey from the North Sea.

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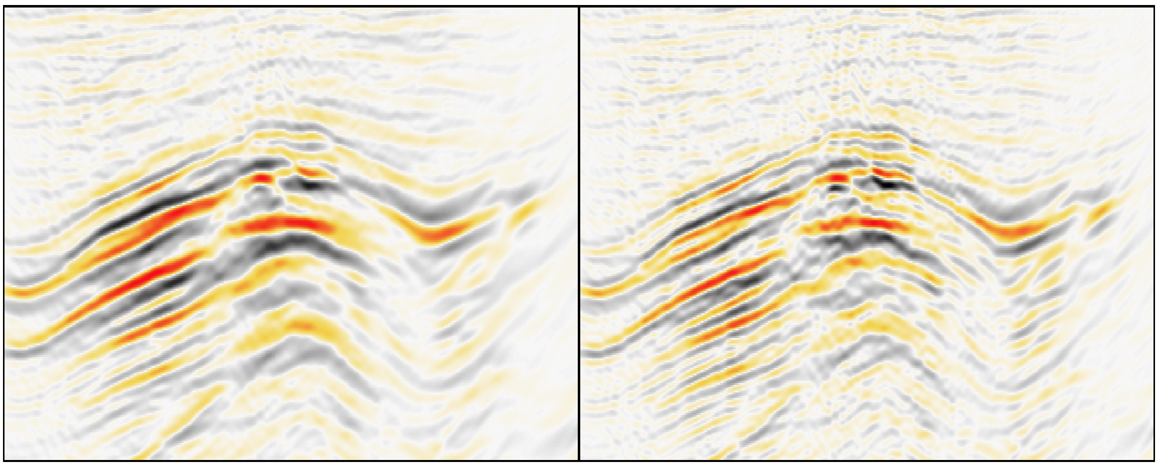

Figure 2. RTM Migrated images: without Q compensation (left) and with Q compensation (right).

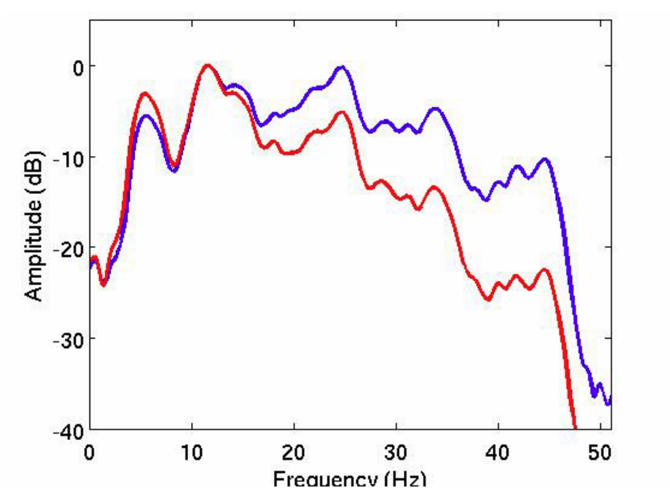

Figure 3. Amplitude spectra for the RTM images (shown in Figure 2) after depth to time conversion: without (red) and with (blue) Q-compensation.