Recovering the Reflectivity Matrix and Angle-dependent Plane-wave Reflection Coefficients from Imaging of Multiples

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SUMMARY

A joint migration approach employing the complete wavefield (containing primary and multiple reflections) requires properly imaged multiples of all orders. By solving a Fredholm integral equation of the first kind, we compute the reflected impulse response (e.g., reflectivity) matrix at every image level and extract the angle-dependent plane wave reflection coefficients. We tested this imaging technique employing a simple 1D layered model with multiple reflections included. We obtained the correct Amplitude Versus Angle (AVA) response of the estimated plane wave reflection coefficient. By simultaneously imaging primaries and multiples, we also achieved an overall better image.

Introduction

In seismic acquisition primary and multiple reflections are simultaneously recorded. Historically, only primaries are used in depth migration, while multiples are treated as undesired noise. Notwithstanding, it has long been recognized that if multiples are handled in a correct way, they provide additional structural information, leading to enhanced subsurface illumination and increased image resolution (e.g., Berkhout, 1985). A joint migration approach employing the complete wavefield (containing primary and multiple reflections) requires properly imaged multiples of all orders. This again demands that the reflected component of the impulse response (i.e., reflectivity) is estimated correctly at every image level.

It is well known from the literature that the reflected component of the impulse response can be recovered from a Fredholm integral equation of the first kind defined in the frequency-space domain (see e.g. Amundsen, 2001; Ordoñez and Söllner, 2013). From this formulation, it follows that the upgoing pressure wavefield (measured after redatuming the receivers from acquisition level to image level) can be synthesized by forming all stationary combinations of the downgoing branch of a filtered version of the vertical velocity wavefield and the reflectivity measured at the same image level, in the special case where the overburden is a homogeneous half-space. Thus, this reflectivity is the after sought quantity at each image level. Assuming that sufficient data are available, an inversion of the matrix form of this integral problem at every image level, gives an estimate of the reflectivity, where ray path interactions (i.e., crosstalk) caused by the physical overburden are eliminated. The matrix solution of this integral problem in the frequency-space domain has already been discussed by several authors (see e.g. Berkhout, 1985; de Bruin et al., 1990). Both zero-offset and angle-dependent responses of the subsurface can be computed by choosing a convenient subset of the reflectivity matrix.

Considering separated wavefields and multiple reflections as part of the signal space, we present a controlled study to illustrate how multiples can be employed to estimate the reflectivity matrix, as well as to extract the angle-dependent plane wave reflection coefficients to create angle gathers. Two different prestack migration experiments were carried out. We used source and receiver wavefields which were solely composed of multiple reflections and we also employed complete wavefields including both primary and multiple reflections.

Method

At every image level, the reflectivity can be extracted from the following Fredholm integral equation of the first kind in the frequency-space domain (Amundsen, 2001; Ordoñez and Söllner, 2013):

$$U^{(P)}(\mathbf{x}_{\mathrm{r}}, \mathbf{x}_{\mathrm{s}}) = -2i\omega\rho \iint_{\partial \mathrm{V}_1} R^{IR}(\mathbf{x}_{\mathrm{r}}, \mathbf{x}) D^{(V_z)}(\mathbf{x}, \mathbf{x}_{\mathrm{s}}) \mathrm{dS}\,, \tag{1}$$

where $U^{(P)}$ represents the upgoing pressure wavefield with the source at position $\mathbf{x}_{\mathrm{s}}$ and recorded at $\mathbf{x}_{\mathrm{r}}$, R^{IR} is the common-receiver gather of the reflectivity (i.e. reflected impulse response) recorded at $\mathbf{x}_{\mathbf{r}}$ and with virtual sources placed along the image level at $\mathbf{x}$, and $D^{(V_z)}$ is the common-source gather of the downgoing vertical velocity wavefield generated at $\mathbf{x}_{\mathbf{s}}$ and recorded by virtual receivers along the same image level at $\mathbf{x}$. Moreover, i represents the imaginary unit, ω is the angular frequency and ρ is the mass density. In equation 1, we have used the Fourier convention: $H(\omega) = \int h(t)\mathrm{e}^{i\omega t}dt$.

The integral problem formulated in equation 1 may be recast into a matrix representation (Berkhout, 1985; de Bruin et al., 1990). By introducing the filtered downgoing vertical velocity$D_f^{(V_z)} = -2i\omega\rho D^{(V_z)}$, we define the matrices $\mathbf{R^{IR}}$, $\mathbf{U^{(P)}}$ and $\mathbf{D_f^{(V_z)}}$, such as their rows correspond respectively to $R^{IR}(\mathbf{x_r}, \mathbf{x})$, $U^{(P)}(\mathbf{x_r}, \mathbf{x_s})$ and $D_f^{(V_z)}(\mathbf{x}, \mathbf{x_s})$ for a fixed source and variable receiver location and their columns represent the reciprocal case. Then, the integral equation 1 becomes:

$$\mathbf{U^{(P)}} = \mathbf{R^{IR}}\, \mathbf{D_f^{(V_z)}}. \qquad (2)$$

Once the reflectivity matrix has been computed, a depth image of the subsurface can be calculated by choosing a subset of the matrix. Most one-way wave-equation migration techniques calculate only zero-offset responses. This corresponds to choosing the principal diagonal of $\mathbf{R^{IR}}$ (Figure 1), representing the case of coinciding virtual sources and receivers in space. We then sum over frequencies to ensure the zero-time lag condition in the time-space domain. By considering other subsets of the reflected impulse response matrix, one can build common source, receiver or midpoint responses (Figure 1).

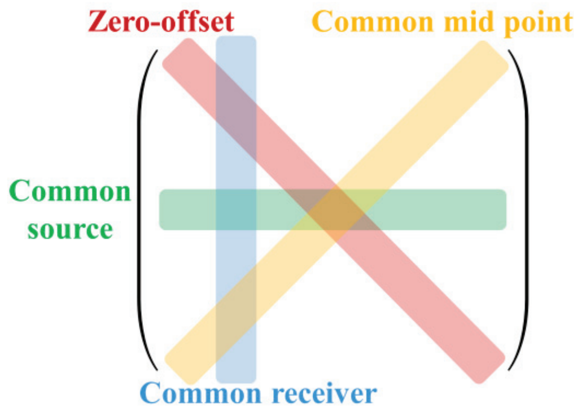

Figure 1 *Structure of the reflectivity matrix*

In the following example, we first determined $\mathbf{R^{IR}}$ by inversion of equation 2. Then, we extracted the diagonal of the reflectivity matrix to build the zero-offset depth image. Next, we selected a row of the reflectivity matrix to extract the plane wave reflection coefficients and build a common source response in the slowness domain (i.e. angle gather). We started by Fourier transforming in the space domain so that $R^{IR}(x,\omega)$ was transformed into $\widetilde{R^{IR}}(k_x,\omega)$, where k_x is the horizontal wavenumber. Since we are after the plane-wave reflection coefficient $\widetilde{R^{PW}}(k_x,\omega)$ (defined as the ratio between the reflected/upgoing pressure and the incident/downgoing pressure), $\widetilde{R^{IR}}(k_x,\omega)$ need to be scaled by the downgoing wavefield created by a point source decomposed into plane waves. Following de Bruin et al. (1990), we constructed an angle gather by mapping the plane-wave reflection coefficient from the wavenumber domain (k_x,ω) to the slowness domain $\left(p_x = k_x/\omega, \omega\right)$, before summing over frequencies. Note that the horizontal slowness is related to the angle $\left(p_x = \sin\alpha/c\right)$.

Example

In our tests, we considered the simple 2D model presented in Table 1.The medium was composed of three layers with the two horizontal reflectors located at a depth of respectively 80 m and 200 m. For both of the two prestack migration experiments considered , we assumed a streamer depth of 25 m and generated controlled data employing a reflectivity modelling code. Each source-gather contained 150 traces separated by 12.5 m. The recording time was set to 1.5 s and the sample rate was 4 ms. The seismic source was located at a depth of 10 m and the source wavelet was a minimum phase Ricker wavelet with a central frequency of 15 Hz. A total of 150 shots were considered for the migrations.

Layer thickness (m)	Velocity (m/s)	Density (g/cm^3)
80	1500	1
200	1600	2
-	1700	2.4

***Table 1** Layered model used to generate the data*

Figure 2 displays the zero-offset image and the angle gather obtained after migration of multiples. Figure 3 displays the migration results obtained when considering primaries and multiples simultaneously. Note that there is a very good match between the depth images of the migrated multiples (Figures 2) and the simultaneously migrated primaries and multiples (Figure 3).

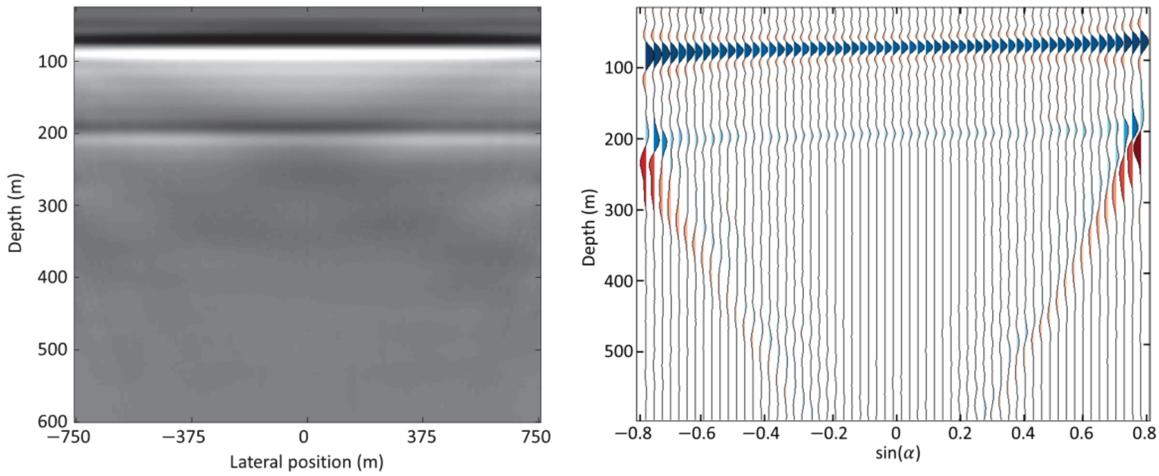

***Figure 2** Zero-offset image (left) and angle gather (right) from migration of multiples only*

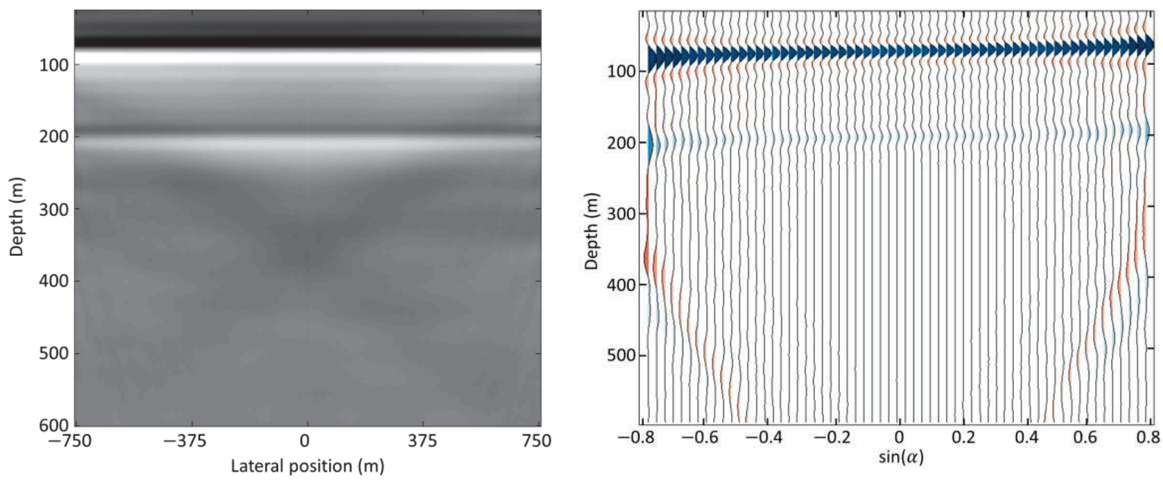

***Figure 3** Zero-offset image (left) and angle gather (right) from migration of primaries and multiples*

Figure 4 shows the amplitude picks extracted along the two events in the angle gathers of Figures 2 and 3 corresponding to the two reflector depths of 80 m and 200 m. We have also superimposed the theoretical Amplitude Versus Angle response derived from the Zoeppritz equations (Aki and Richards, 1980). For the first event (Figure 4-left), the derived responses from migration of multiples only and simultaneous migration of primaries and multiples match the theoretical response up to respectively 64 degrees ($\sin\alpha=0.9$) and 72 degrees ($\sin\alpha=0.95$). For the second event (Figure 4-

right), the match is acceptable up to 58 degrees ($\sin\alpha$=0.85). The deviations observed at higher angles are due to the limited aperture.

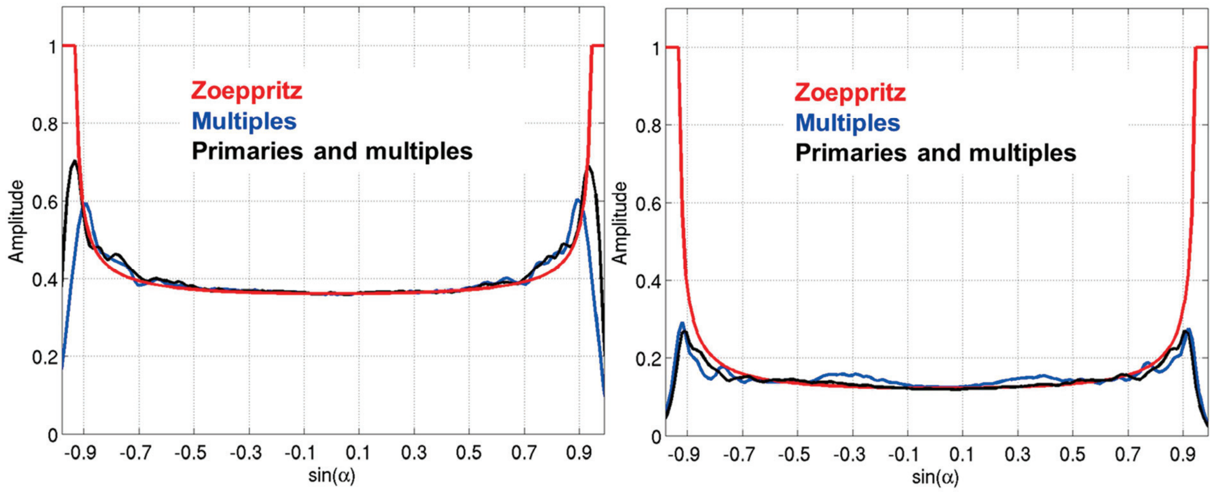

Figure 4 *Amplitude picks extracted along the events at 80 m/left) and 200 m (right)*

Conclusions

Before migration of primaries and multiples can be carried out in one step, we need to develop an imaging technique that ensures properly migrated amplitudes for all angles. In case of a simple synthetic model and considering multiples reflections as part of the signal, we computed the reflectivity matrix. By extracting particular subsets of this matrix, we demonstrated the feasibility of forming both zero-offset images and angle gathers. Indeed, the migration of multiple reflections led to a correct AVA response. We also imaged primaries and multiples simultaneously and computed the same type of responses. The use of the complete data set gave rise to overall better results.

Acknowledgements

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References

Aki, K., and P.G. Richards, 1980, Quantitative seismology: W.H. Freeman and Co.

Amundsen, L., 2001, Elimination of free-surface related multiples without need of the source wavelet: Geophysics, **66**, 327-341.

Berkhout, A.J., 1985, Seismic migration: Imaging of acoustic energy by wave field extrapolation: Elsevier Science Publ. Co., Inc.

de Bruin, C. G. M., C. P. A. Wapenaar, and A. J. Berkhout, 1990, Angle-dependent reflectivity by means of prestack-migration: Geophysics, **55**, 1223–1234.

Ordoñez, A., and W. Söllner, 2013, Imaging condition for pressure-normalized separated wavefields: 75th Annual Conference and Exhibition Meeting, EAGE, Extended Abstracts, We 02 04.