

Extraction of the Normal Component of the Particle Velocity for Arbitrarily Shaped Surfaces

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SUMMARY

The normal component of the particle velocity (or the normal derivative of the pressure field) at the recording surface is an indispensable information when deghosting or extrapolating marine seismic data. However, the pressure field at the recording surface is related to the normal component of the particle velocity based on Kirchhoff-Helmholtz integral equation. Nevertheless, extracting the normal component of the particle velocity from the recorded pressure is not a trivial task. This is because; first, the Green's function containing the scattering information from a spatio-temporally varying sea surface must be known; and second, the signal-to-noise ratio of the pressure measurement at the notch frequencies are notoriously poor. We present a method based on Kirchhoff-Helmholtz integral equation for extracting the normal component of the particle velocity (away from the pressure notch locations) for any arbitrarily shaped sea surfaces. The validity of the method is demonstrated using both synthetic and field datasets obtained using a dual-sensor streamer.

Introduction

Extrapolation or deghosting of the total pressure field based on Kirchhoff-Helmholtz integral equation requires the pressure and the normal component of the particle velocity at the recording surface as input. However, these two input data are not prescribed independently. Thus, provided that we are away from the notch locations and know the shape of the sea surface, the normal component of the particle velocity can be determined from the recorded pressure. Amundsen et al. (1995) presented a method based on the Kirchhoff-Helmholtz integral equation for extracting the normal component of the particle velocity at the recording surface for the case of a flat sea surface. In this paper, we generalize this idea for any arbitrarily shaped sea surfaces. Information about the shape of the sea surface at any given space and time can be obtained employing dual sensor data (Orji et al., 2010) or from very low frequency pressure recordings (Laws and Kragh, 2006). The generalized scheme for extracting the normal component of the particle velocity is implemented and validated using rough sea synthetic data and deep water field data from offshore Brazil.

Methodology

Consider a time harmonic pressure field, $p(\mathbf{r}_r, \mathbf{r}_s)$ generated by a source at \mathbf{r}_s and recorded at a receiver \mathbf{r}_r . This pressure field satisfies the homogenous Helmholtz wave equation. In marine seismic acquisition, a suitable closed surface S that has the recording surface S_r is often selected (cf. Fig. 1a). Using the equation of motion to relate the normal component of the particle velocity with the normal derivative of the pressure field and, selecting a causal Green's function, with the same medium parameters as the pressure field within and on the surface S , the Kirchhoff-Helmholtz integral equation can be written as (Morse and Feshbach, 1953)

$$\alpha p(\mathbf{r}) = \int_{S_r} \left[-i\omega\rho G(\mathbf{r}_r, \mathbf{r})v_n(\mathbf{r}_r, \mathbf{r}_s) - p(\mathbf{r}_r, \mathbf{r}_s) \frac{\partial G(\mathbf{r}_r, \mathbf{r})}{\partial n} \right] dS_r \quad (1)$$

Where,

$$\alpha = \begin{cases} 1, & \text{if } \mathbf{r} \text{ is inside } S \\ 0, & \text{if } \mathbf{r} \text{ is outside } S, \end{cases}$$

with ω and ρ denoting the angular frequency and density of the medium, respectively. Moreover, $\frac{\partial}{\partial n}$ signifies a partial derivative in the outward normal direction and i represents $\sqrt{-1}$. Here, we have employed the fact that the closed surface S can be decomposed into the recording surface S_r and a hemispherical surface S_R . By assuming the surface S_R is located at infinity, its contribution to the Kirchhoff-Helmholtz integral equation becomes zero as a result of Sommerfeld radiation condition (Sommerfeld, 1954).

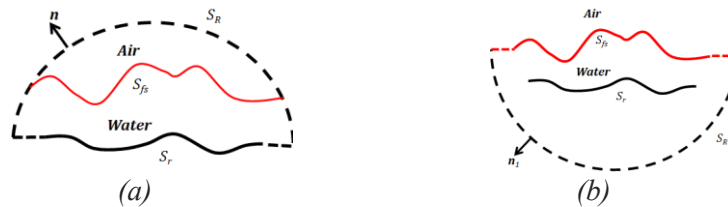


Figure 1 Problem geometry with the closed surface S comprising (a) the recording surface S_r and a hemispherical surface S_R . (b) the free surface S_{fs} and a hemispherical surface S_R .

To relate the recorded pressure with the normal component of the particle velocity, we consider the Green's function source location lies below the recording surface ($\mathbf{r} = \mathbf{r}_r + \boldsymbol{\varepsilon}$, with $\boldsymbol{\varepsilon}$ being some distance in the normal direction) and replacing the integration with that of quadrature summation, Eq. 1 reduces to

$$\mathbf{D}_{qj} p^j = -i\omega\rho \mathbf{M}_{qj} v_n^j, \quad (2)$$

\mathbf{D} and \mathbf{M} are the *dipole* and *monopole* matrices, respectively. q is the Green's function source index and j is the receiver location index. Here summation is implied over all repeated indices. It is pertinent to note that selecting a very small or very large $\boldsymbol{\varepsilon}$ result in numerical artifacts during the extraction of the normal component of the particle velocity (Amundsen et al., 1995).

Until now we have assumed that the Green's function and its normal derivative are known (or \mathbf{D} and \mathbf{M} are known). When the sea surface is flat, the Green's function and its normal derivative can be obtained using *the method of images* (Morse and Feshbach, 1953). However, when the sea surface varies in shape (or does not correspond to separable coordinate system geometries), the Green's function can be determined based on the Kirchhoff-Helmholtz integral equation with the *actual source inside* the closed surface. Consider now a closed surface that includes the free surface (cf. Fig. 1b) and replace the actual source with that of a Dirac delta pulse $\delta(\mathbf{r}' - \mathbf{r})$. Invoking Sommerfeld radiation condition over S_R and imposing the free surface boundary condition over S_{fs} (or setting $G(\mathbf{r}')|_{r^s} = 0$, where \mathbf{r}^s is the observation point on S_{fs}); the Kirchhoff-Helmholtz integral equation gives

$$\alpha G(\mathbf{r}') = G^0(\mathbf{r}, \mathbf{r}') + \int_{S_{fs}} \left[G^0(\mathbf{r}^s, \mathbf{r}') \frac{\partial G(\mathbf{r}^s, \mathbf{r}')}{\partial n_1} \right] dS_{fs}, \quad (3)$$

Where

$$\alpha = \begin{cases} 1, & \text{if } \mathbf{r}' \text{ is inside } S \\ 0, & \text{if } \mathbf{r}' \text{ is outside } S, \end{cases}$$

G^0 is the free space Green's function and where n_1 is the normal at the surface. Here, note that the Green's function $G(\mathbf{r}')$ represents the pressure field as a result of two sources; first the actual Dirac delta pulse and second the free surface. The strategy for computing the Green's function and its normal derivative at the streamer location based on Eq. 3 is summarized as follows: (i) Selecting $\alpha = 1$ in Eq. 3 and taking the limit when \mathbf{r}' approaches the free surface and also using the free surface boundary condition ($G(\mathbf{r}')|_{r^s} = 0$), calculate the normal derivative of the Green's function at the free surface, (ii) Inserting back the computed normal derivative of the Green's function at the free surface from (i) and solving Eq. 3 for $\alpha = 1$ and $\mathbf{r}' = \mathbf{r}_r$, the Green's function at the streamer location is computed, and (iii) the normal derivative of the Green's function at the streamer location can be obtained by taking the normal derivative of the result in (ii). This is mathematically written as

$$\frac{\partial G(\mathbf{r}_r)}{\partial n} = \frac{\partial G^0(\mathbf{r}, \mathbf{r}_r)}{\partial n} + \int_{S_{fs}} \left[\frac{\partial G^0(\mathbf{r}^s, \mathbf{r}_r)}{\partial n} \frac{\partial G(\mathbf{r}^s, \mathbf{r}')}{\partial n_1} \right] dS_{fs}. \quad (4)$$

Synthetic Data Example

A 2D pressure and normal component of the particle velocity were modeled using the integral method for a model consisting of rough sea surface and half space of water. The rough sea surface is based on a Pierson-Moskowitz spectrum with a wind speed of 15 m/s. A source at 1 km depth and receivers at 7.5 m were used to compute the data. The data were generated using a temporal and spatial sampling of 4 ms and 6 m, respectively. Figures 2(a) and (b) show the modeled pressure and normal component of the particle velocity, respectively. Their corresponding amplitude spectra are shown in Figs. 2(c) and (d). The amplitude spectra show that the receiver ghost notches are random; this is because the sea surface is very rough (having a root-mean-square wave height of 1.7 m).

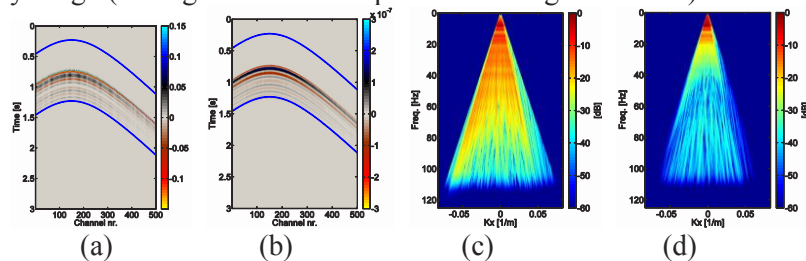


Figure 2 (a) Pressure wavefield, (b) normal component of the particle velocity, (c) amplitude spectrum of (a), and (d) amplitude spectrum of (b).

In order to better illustrate the behavior of the notches for rough sea, Figs. 3(a) and (b) show the amplitude and phase spectra of the ghost function calculated at vertical incidence, respectively. As a consequence of the rough sea surface, the following can be observed from these figures; (i) below 20 Hz the effect of rough sea is negligible, (ii) the second notch location is different from a frequency location that would have been predicted by a flat sea surface, (iii) the effect of incoherent scattering is stronger at higher frequencies, (iv) the signal level present at the notch location is small (or

alternatively poor signal-to-noise ratio for field data). An attempt to deghost at this location would result in instability (or noise amplification for field data), and (v) the phase of the ghost function undergoes a sharp change at the notch location, which is different from that predicted by a flat sea approximation.

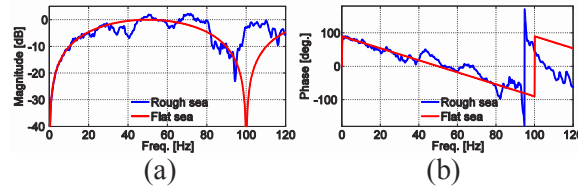


Figure 3 (a) Amplitude and (b) phase spectra of rough (blue) and flat (red) sea surface ghost functions at the vertical incidence.

Sea surface imaging was performed within a 1 s window following the event (cf. Fig. 2(a)) for different frequency bands (cf. Fig. 4(a)). The imaged sea surfaces for the bandwidth 0–125 Hz, 20–125 Hz and 40–125 Hz show very similar results. This is confirmed by comparing the results with the true sea surface. However, the effect of a band limited source and a limited aperture is still present. These negligible effects can easily be handled in practice.

The second step in the extraction of the normal component of the particle velocity is modeling of Green’s functions and their corresponding normal derivatives (or the construction of the *monopole* and *dipole* matrices). Placing the Green’s function source 10 m below the streamer, we generated the *monopole* and *dipole* matrices for Eq. 2. Here, the location of the Green’s function source was selected after testing different locations and selecting that with the smallest artifact.

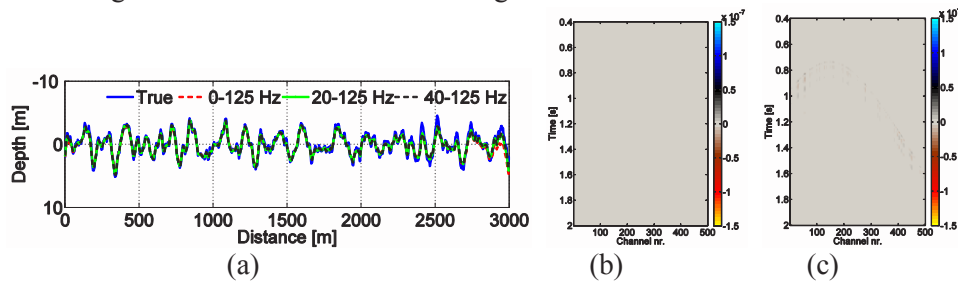


Figure 4 (a) True and imaged sea surfaces obtained using different frequency bandwidths, (b) the residual between the modeled and extracted normal component of the particle velocity based on Eq. 2, assuming the sea surface shape is known exactly, (c) using the imaged sea surface. NB: For the purpose of visualization, (b) and (c) are shown with half the color scale relative to that used for modeled data in Fig. 2(b).

Finally, the normal component of the particle velocity is extracted by solving an even-determined inverse problem based on Eq. 2. To avoid the receiver notch locations and possible instabilities associated with being close to the vicinity of these notches, the analysis was limited to the frequency band between 0 and 55 Hz. Firstly, assuming the sea surface shape is known exactly, we computed the residual between the modeled and extracted normal component of the particle velocity as shown in Fig. 4(b). This residual difference is negligible implying that the method correctly predicted the normal component of the particle velocity. Secondly, we used the imaged sea surface and again extracted the normal component of the particle velocity and then computed the residual between the modeled and extracted as shown in Fig. 4(c). Here, the residue is related to the negligible differences between the modeled and imaged sea surfaces.

Field Data Example

The field data was acquired by PGS using dual-sensor streamer in deep water offshore Brazil. The sources and receivers were at a depth of 7 m and 15 m, respectively. Sea surface imaging was performed using 20–25 Hz highpass filtered up- and down-going pressure wavefields within a 1 s window around the sea floor primary reflection event. The peak to peak wave height estimated from

the imaged sea surface is around 3.6 m, which matches the observers wave height estimate of 3.2 – 3.8 m (cf. Fig. 5(a)).

Employing the imaged sea surface and following the same approach as in the synthetic data section, the normal component of the particle velocity was extracted. Since the data is from deep water, we utilized a 2D Green's function for the construction of the *monopole* and *dipole* matrices. Figures 5(b) and (c) respectively show the extracted normal component of the particle velocity and its amplitude spectrum for the time window following the primary reflection of the sea floor event. The validity of the results was confirmed by computing the residual between the extracted and measured normal component of the particle velocity within the frequency band between 15 and 45 Hz (cf. Fig. 5(d)). The negligible residue confirms that the extraction of the normal component of the particle velocity was successful. Nevertheless, the minor differences can be attributed to possible discrepancies between the imaged and the true sea surface as demonstrated using synthetic data.

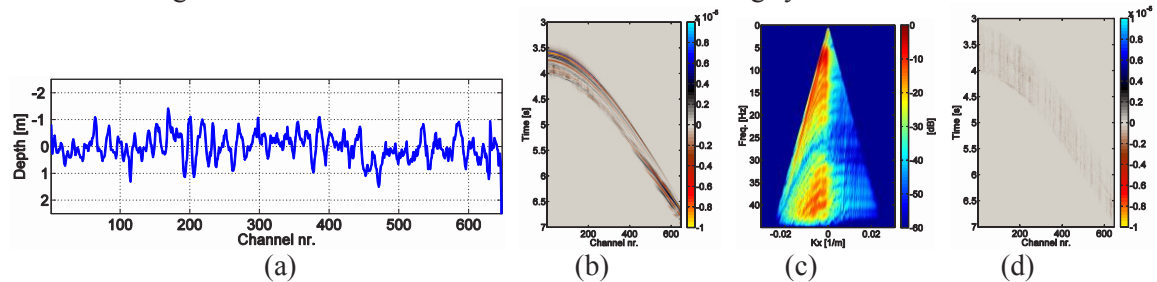


Figure 5 (a) The imaged sea surface for the event following the primary reflection, (b) extracted normal component of the particle velocity for the primary event, (c) amplitude spectrum of (b), and (d) the residual between the measured and extracted normal component of the particle velocity.

Conclusions

A method based on Kirchhoff-Helmholtz integral equation for extracting the normal component of the particle velocity at the recording surface for any arbitrarily shaped sea surface is proposed. The method requires the Green's function that contains the scattering information from the spatiotemporally varying sea surface. This is achieved by first imaging the sea surface employing dual sensor data for the frequency band with high signal-to-noise ratio (i.e. high frequencies) and then computing the Green's function utilizing the Kirchhoff-Helmholtz integral equation for a closed surface with an actual source that is a Dirac delta pulse in space. The validity of the technique has been demonstrated using a rough sea synthetic data modeled employing the integral method and deep water field data from offshore Brazil. In both tests, a successful extraction of the normal component of the particle velocity was obtained.

Acknowledgements

We thank PGS for permission to publish this work. We are grateful to our colleagues Hocine Tabti, Tilman Klüber and Anthony Day for fruitful discussions.

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