Modeling close range air gun interactions using isosurfaces of the velocity potential

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SUMMARY

We describe an alternative way of modeling clustered air guns, by allowing the shape of the air bubbles to be described by isosurfaces of an incompressible velocity potential, as opposed to the traditional way of considering them as perfect spheres. This method solves some close-range interaction problems, and may possibly be used as a basis for a more complete modeling of clustered air guns.

INTRODUCTION

Clustering of air guns, i.e. the positioning of two or more air guns in such close proximity to each other that the air bubbles will coalesce, is a popular way of improving the broadband signature from an air gun array. The notable effects of clustering are an increased bubble time period which will increase the low frequency output, an improved primary-to-bubble ratio, and a slight drop in the primary amplitude compared to firing the guns at a greater separation distance.

When designing broadband marine seismic sources, for which the demand is increasing (Laws et al. (2008); Hegna and Parkes (2011); Kragh et al. (2012)), it is important to be able to model the sources correctly in order to optimise the output. Typically, this can be done with the required accuracy for single gun arrays by modeling the set of single guns as spherical air bubbles, and applying a correction to the hydrostatic pressure they experience at a certain time by adding the combined emitted wavefield from all the other guns, and their ghost signals (Ziolkowski et al.,1982). However, this method becomes problematic when two or more of the spheres will overlap, as there is no intuitive way of handling the wavefield propagation between the two correctly. An additional factor is the small time delays of the emitted signal, which may cause numerical problems for the differential equation solvers.

One way of avoiding this problem is to treat the interaction as working in an incompressible fluid, as this will immediately resolve the timing issues for the numerical solver since only the current state is needed at each time step. Since the distance between the two bubbles is so small anyway, it should not introduce large errors. This approach is used for more complex modeling schemes, such as the ones based on boundary integral models (see for instance Cox et al. (2004)), which solve for the entire bubble shape at each time step. Such models do, however, tend to be quite computationally expensive and are often unstable, and are therefore unsuitable for using modeling for optimization of air gun arrays.

At the same time, a spherical compressible model has been shown by Barker and Landrø (2012) to be able to describe the relative bubble time period of non-coalescing bubbles by a energy comparison, so even simpler incompressible models seem to contain the vital parts of physics concerning the increase of the bubble time period.

A good approximation may then be to allow the air bubble shape to deviate from a perfect sphere, and merge with other bubbles, but restrict the number of possible shapes it may obtain in order to reduce the number of variables used by the solver.

One possible family of shapes which can be used are the isosurfaces of the velocity potential, which we will investigate here.

THE GENERALISED RAYLEIGH EQUATION

The simplest differential equation trying to describe the motion of a submerged bubble filled with a gas is the equation described by Rayleigh (1917). It is based on the assumption of a spherical bubble in an incompressible fluid, has no energy loss and can be derived by considering an energy balance between the kinetic energy of the fluid and the potential energy of the bubble (see for instance Barker and Landrø (2012)). It is possible to extend the equation to be valid for non-spherical bubbles as well, as long as it is possible to calculate the corresponding kinetic energy. The potential energy is a function of the volume and equation of state, and is therefore unchanged in this case. By assuming that the energy is proportional to the corresponding energy of a spherical bubble of the same radius, R, such that

$$E_k = f \cdot 2\rho\pi R^3 \dot{R}^2, \tag{1}$$

where ρ is the fluid density, and a dot denotes differentiating with respect to time, we can generalise the Rayleigh equation to

$$\ddot{R} = \frac{1}{f}\left(\frac{P - P_\infty}{\rho R}\right) - \frac{3}{2}\frac{\dot{R}^2}{R} - \frac{\dot{R}}{2f}\frac{\mathrm{D}f}{\mathrm{D}t}, \tag{2}$$

where $f = 1$ describes the normal Rayleigh equation. While the Rayleigh equation itself is based on an incompressible fluid, which would not allow any acoustic signal to be emitted, we can still estimate such a signal by evaluating the Bernoulli equation at a retarded time, such that the farfield signal at position Q is given by (Cox et al., 2004)

$$P(r,t) = P_{\infty,Q} - \rho \left.\frac{\partial \phi}{\partial t}\right|_{t-\frac{r}{c},Q} \tag{3}$$

ISOSURFACE-SHAPED BUBBLES

In order to be able to handle coalescing bubbles, we need a smooth model for how the bubbles will be shaped, both when apart and when they have merged to one bubble. There are, of course many ways to create such a family of shapes, but it seems logical that they should at least satisfy two basic cases. When the separation distance is infinite, the bubbles should be spherical like we would expect a single bubble to be, and when the separation distance is zero we should see one spherical bubble with a volume equaling the combined volume of two single bubbles. One way of doing this is to let the shapes be isosurfaces of the velocity potential, ϕ, as is used by Barker and Landrø (2013) to make a simple estimate of the bubble time periods of air guns without having to solve any differential equation for the bubble wall motion. This means that the surface is described as any point in the valid domain where $\phi = C$, for some constant C which has to be picked such that the surface contains the correct volume. While this is not the only possible way of describing the bubble surfaces, it conveniently makes the particle velocity at the bubble wall to be normal to the wall itself, which is usually only approximated by the introduction of corrections to the potential (Cox et al., 2004).

However, using isosurfaces to describe the bubbles has a downside as well, which is the fact that the shapes themselves are not easily described analytically (except for the two-bubble case) and needs to be found numerically. To find the surface for a given velocity potential, we then need to seek for the constant value C from the bubble center out in enough directions to adequately describe the shape, for instance by some spline interpolation. Since the constant itself can not be given analytically, this will need to be found as well, so for a given volume V we have to

1. Guess the value of C.
2. Determine the bubble surface by numerical seek in 3 dimensions (or fewer, if we have some symmetry).
3. Calculate the volume inside the surface.
4. Adjust our guess for C and return to step 2 if the volume is not sufficiently close to V.

where the whole process is improved by running a standard optimization routine. Figure 1 shows a set of general isosurfaces for a two bubble system, described by

$$\phi = -\frac{\dot{V}}{4\pi r} - \frac{\dot{V}}{4\pi\sqrt{r^2 + 4b^2 + 4br\cos(\theta)}}, \qquad (4)$$

where the origin is placed in the center of the rightmost bubble, (r, θ) are the parameters used in a spherical coordinate system, $\dot{V}$ the volume derivative and $2b$ is the separation distance between the two centers.

However, it can be shown that the shape of the surfaces can be generalised as a function of separation distance divided by radius (in the sense of $R = \left(\frac{3V}{4\pi}\right)^{\frac{1}{3}}$), which coincides with the discovery made by Strandenes and Vaage (1992) that most interaction effects in air gun clusters depend on a similar parameter, the only difference being that the radius they use is a specific equilibrium radius, which is the radius at which the air inside the bubble has hydrostatic pressure and a temperature equal to the surrounding fluid.

This means that for a given volume, we can use a unit isosurface, and just scale that surface up to the desired volume.

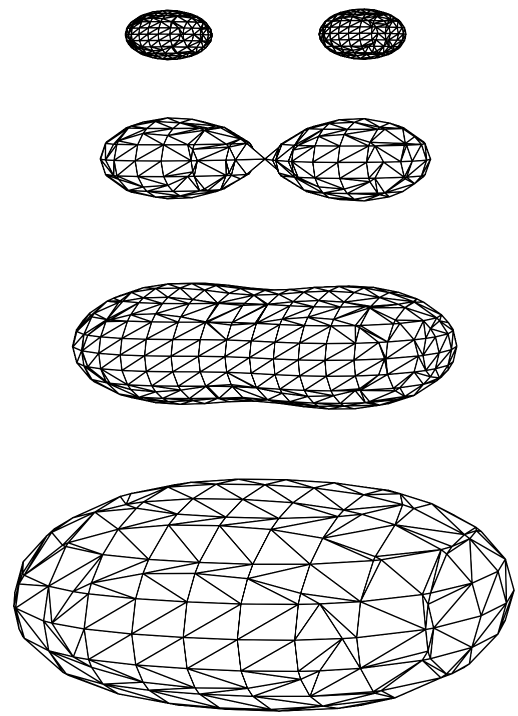

Figure 1: A set of isosurfaces with a fixed separation distance and differing volumes, displaying how the bubbles may merge with this representation.

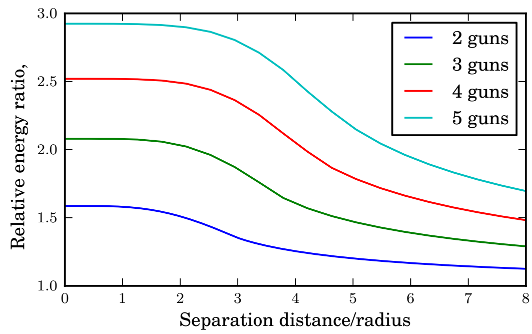

Figure 2: Calculated energy curves, as function of separation distance divided by radius, for 2-5 guns in a circular configuration.

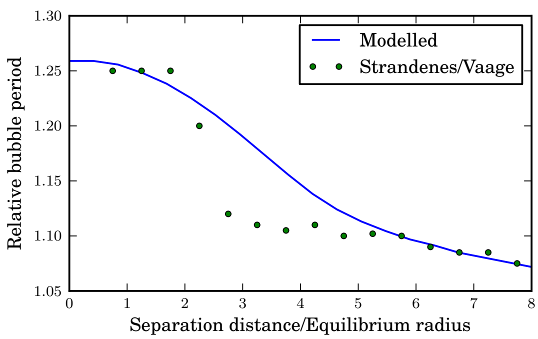

Figure 3: Modeled relative bubble time period, along with measured data from Strandenes and Vaage (1992).

Combining this with the fact that the energy relative to a single bubble can be calculated as a function of the constant C and the volume itself (Barker and Landrø,2013), it is possible to pre-calculate the energy function required by Equation 1 at specific volumes. By interpolating between these points, it is then possible to do air bubble modeling with no significant increase in the computation needed at run-time. Examples of these energy curves for 2-5 guns in a circular configuration is shown in Figure 2. The energy at infinite separation distance asymptotically approaches the corresponding energy of a single gun, and the energy at zero separation distance for n guns is equal to one part in n of the energy of a gun with n times the volume (the other parts of the energy being assigned to the other bubbles). For the two bubble case, doubling the volume will mean that the radius will be $2^{\frac{1}{3}}$ the radius of a single bubble and by combining with Equation 1, we have

$$E_2 = \frac{1}{2} \cdot 2\rho\pi \left(2^{\frac{1}{3}}R\right)^3 \left(2^{\frac{1}{3}}\dot{R}\right)^2 = 2^{\frac{2}{3}}E_1, \qquad (5)$$

and likewise, for an n bubble case $E_n = n^{\frac{2}{3}}E_1$.

Figure 3 shows the modeled relative bubble time period, calculated by using a Runge-Kutta 4(5) order ordinary differential equation solver (Hairer et al., 2009) on Equation 2 using adiabatic expansion, compared with the experimental data-points presented by Strandenes and Vaage (1992). An example of the

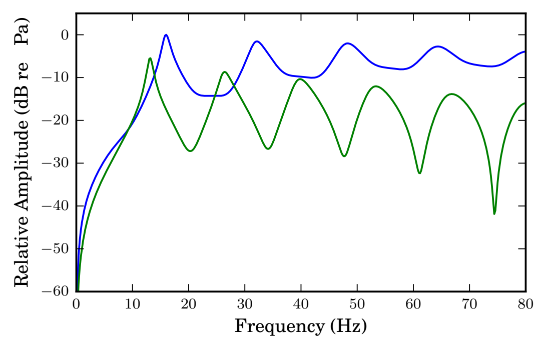

Figure 4: The modeled effect of clustering to the spectrum. A cluster consisting of two 40 cu.in. guns is compared to the sum of two single 40 cu.in. guns. Both cases are at 5m depth.

effect on the spectrum is shown in Figure 4, where we have compared a 40 cu.in. gun with a cluster containing 2 such guns at a separation distance of 0.5 m, which is approximately 2 times the equilibrium radius. To get a realistic spectrum, a mass transfer effect as described by Langhammer (1994) was added to remove the infinite oscillation. We see that the first low-frequency peak of the cluster is shifted to the left, and that the maximum frequency of the single gun is approximately 1.23 times that of the cluster, which coincides with the appropriate ratio (2) in Figure 3.

DISCUSSION

From Figure 3 we see that we are able to obtain reasonable bubble time period variations when modeling very small, and even zero, separation distances, and the limit at zero is the same as we would expect from one bubble with n times the volume. It is also evident that the modeled relative bubble time period is not quite as accurate when the separation distance divide by equilibrium radius is around 3. Since this is approximately the ratio where bubbles will coalesce during the expansion phase, there are several reasons that might cause greater error here.

First of all the separation distance in the modeling is kept constant. While the separation of the air guns will be constant when firing, the distance between the centers of the bubbles usually are not. During the first expansion phase, the two bubbles will typically move slightly further apart due to the expansion of the other bubble, and then the contraction will cause the two bubbles to merge into one, and stays as one bubble for the rest of the oscillations. Since the bubbles will be pushed slightly away from each other, the lack of movement of the bubbles might cause the modeling to behave as if the separation distance is smaller than it actually is during this phase, which will cause the relative time period to increase and explain why it is overestimated for for values of separation distance divided by equilibrium radius above 2.5.

Correcting for this by incorporating movement, could then be believed to underestimate the relative bubble time period for smaller separations. With respect to this, we should note that since the Rayleigh equation will create an infinitely oscillating set of bubbles, bubbles which may have merged during expansion might be separated again during contraction. This is clearly an unphysical result, which would also be removed by correctly modeling the bubble movement, and increase the modeled relative bubble time periods for smaller separation distances.

We should also note that when firing an actual air gun, the bubble is not released separate from the gun itself, which will cause a mismatch between radius and volume compared to our model. Correcting for this, by for instance using a bigger radius as function of volume when calculating f, will also increase the estimates for smaller separation distances, although it should still be bounded by $n^{\frac{1}{3}}$.

CONCLUSIONS

By using isosurfaces to represent the bubbles, and using an extended Rayleigh equation, we have shown that we can model air bubbles with very small, and even zero, separation distance and get a smooth transition between two completely separate bubbles and one bubble of the double volume. The modeled results for relative bubble time period seem physical and reasonable compared to data. This smooth transition would not have been possible by using spherical bubbles and a pressure-wavefield interaction.

While isosurfaces are the surface of choice here, it is by no means the only choice, and other choices of surfaces may be better. This is specifically evident, when we look at the shapes used, as they do not look like the bubbles created by an air gun cluster during the first expansion phase, but look more like the shapes of the contracting merged bubble. This could for instance indicate that it might be preferable to increase the allowed set of shapes.

We have not investigated the primary-to-bubble ratio or relative decay of primary amplitude. Modeling the primary-to-bubble ratio requires some sort of energy loss, and the primary decay would require modeling the release of air into the water, neither of which have been done here. Moreover, it seems unlikely that the improved primary-to-bubble ratio experienced when clustering can be adequately modeled without taking into account the movement of the bubble centers toward each other, which requires an even further improved model of bubble dynamics.

It should also not be forgotten that the basic equation here, the Rayleigh equation, is the most basic equation for single bubble dynamics, and that there exist incompressible models which should be more physical. Still, we believe the general notion of using isosurfaces, or other surfaces, in the way described here should be possible to combine with more physical models, and may be helpful in improving the modeling of big air gun clusters.

ACKNOWLEDGEMENTS

We acknowledge PGS for financing the PhD study of Daniel Barker in which most of the work here was done. Martin Landrø acknowledges the financial support from the Norwegian Research Council to the ROSE consortium at NTNU.

http://dx.doi.org/10.1190/segam2013-0489.1

EDITED REFERENCES

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