Taper Design for Block Processing of Seismic Data

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SUMMARY

In many seismic processing applications, such as filtering and interpolation, the input data is partitioned into small overlapping blocks that are processed separately and then merged to construct the final output data. Tapering the different blocks before merging is done to account for the overlap and to avoid visible artefacts related to the imprint of the blocking. While tapering is important, the design of blocking tapers is often done in a simplistic way. This paper proposes a method to design any N-dimension blocking taper with any amount of overlap. The proposed taper has the property that its aggregate sum across the data section is one, thus fold computation and normalization is avoided. In addition, the taper has some useful spectral features so it can also be used as a pre-Fourier transformation taper when the processing involves such transformation.

Introduction

Many seismic applications such as filtering and interpolation involve processing the seismic section in portions, called *blocks*, rather than processing the entire section at once. For example, the de-noise procedure f-x deconvolution (Canales, 1984), works over blocks of data defined by a rectangular time-space window that slides with an overlap in both directions to cover the entire data section.

The main reason for block processing is to fulfil the theoretical modelling assumption behind the processing algorithm. For example, in the case of f-x deconvolution, the seismic event contrary to random noise is supposed to be spatially predictable. This implies that the seismic event is linear with respect to the x variable. However, this assumption is not valid for real data (e.g. reflection events with respect to offset), and can only be approximated locally, i.e., if a small block of data is considered instead.

In all block processing applications, the blocks are extracted with an overlap to achieve a seamless result. The final processing outcome is obtained by merging the result of all processed blocks. To account for the overlap, tapers are applied before merging the different blocks. Tapering is important to avoid visible artefacts related to the imprint of the block size. Linear tapers with an overlap of up to 50% are the default choice of block tapers because of their simplicity and their aggregate sum to unity.

When the processing involves passing the input data through Fourier transforms, the block taper often serves as pre-transformation taper for convenience. Because if a conventional pre-transformation taper such as Hamming or Cosine is used, then one need to make sure to normalize the final output section by the fold, i.e. the aggregate sum of the pre-transformation taper's weights throughout the section (which does not equal to one). This requires computing and buffering the fold and may introduce artefacts due to fold variation. Removing the effect of the pre-transformation taper after the block processing is not done in practice as dividing the output of the block processing by the taper's values is not desirable. Tapers have small values at the edges and the division can cause visible numerical artefacts. Therefore, the idea of using the block taper as a pre-transformation taper is attractive. The design of the block taper should hence be done with some care to account also for its spectral properties. Dimensionality of the taper is not considered in its design. High dimensional tapers are often constructed by simply multiplying a set of one dimensional tapers. This, as it will be shown in an example, may lead to poor shaping of the spectrum of the resulting high-dimensional taper.

The paper proposes a method to design one or multi-dimensional taper with any user's specified amount of overlap. The taper is designed such that (i) its aggregate sum throughout the section is equal to one, hence avoiding fold computation and normalisation, (ii) it satisfies a given smoothness condition that gives it some desirable spectral properties.

Method

The method for designing one dimensional taper is considered first and then generalized to higher dimensional tapers.

Design of 1-d taper

Let $\mathbf{x} = (x_1\ x_2 \ldots x_N)^T$ be a vector that contains the samples of the 1-d taper of length N. The taper moves with M samples which results in an overlap of $L = N - M$ samples. It is assumed for ease of derivation that the taper moves along an infinite axis. The aggregate sum of the taper's weights as it moves along the axis is given by:

$$y_n = \sum_{-\infty}^{\infty} x_{n+kM} \qquad \text{for } 1 \le n + kM \le N \qquad (1)$$

One can easily see that y_n is a periodic sequence with period equal to M. The main idea here is to try to find the taper's samples $(x_1\ x_2 \ldots x_N)$ such that y_n is equal to 1 for one period. This can be formulated as:

$$\begin{aligned} x_1 + x_{M+1} + x_{2M+1} + \cdots &= 1 \\ x_2 + x_{M+2} + x_{2M+2} + \cdots &= 1 \\ \vdots \quad &= 1 \\ x_M + x_{2M} + x_{3M} + \cdots &= 1 \end{aligned} \qquad (2)$$

In practice the taper is moved along a finite length axis. The periodicity, in this case, holds except for the roll-on and roll-off parts which correspond to the edges of the axis. Extending the length of the finite axis by data mirroring at the edges is often done to avoid edges-related artifacts. The above sets of equations are symbolically expressed in a matrix form as:

$$\mathbf{Hx} = \mathbf{b}, \qquad (3)$$

where $\mathbf{H} = [\mathbf{e}_1, \mathbf{e}_2, \cdots \mathbf{e}_M, \mathbf{e}_1, \mathbf{e}_2 \cdots]$ is *M*-by-*N* matrix with $\mathbf{e}_k = (0, \cdots 1, 0, \cdots)^T$ is a vector of *M* "0" element except a "1" at location *k* and $\mathbf{b} = (1, 1, \cdots 1)^T$.
For example, if $(N, M) = (5,2)$ equation (2) becomes:

$$\begin{aligned} x_1 + x_3 + x_5 &= 1 \\ x_2 + x_4 &= 1 \end{aligned} \qquad (4)$$

The linear system in (2) is underdetermined because the number of unknown N (number of samples in the taper) is always larger than the number of equation M (number of samples by which the taper moves). This means that the taper that satisfies equation (2) is not unique. To illustrate this point, all the following tapers are valid solutions to eq. (4)
(0.25, 0.50, 0.50, 0.50, 0.25), (0.10, 0.40, 0.70, 0.60, 0.20), (0.20, 0.60, 0.70, 0.40, 0.10),
The non-uniqueness of the solution is rather a desired property as it gives the designer the flexibility to put additional constraints to shape the taper in order to fit any specific application. In this paper, the emphasis is on enforcing some form of "smoothness" on the taper. The criterion chosen here is to select the taper whose second order difference sequence (i.e. discrete equivalence to the second order derivative) has the minimal energy. This translates to the minimization of a quadratic cost function with the linear constraint in equation (2), i.e.

$$\text{minimize } \|\mathbf{Qx}\|_2^2 \text{ given that } \quad \mathbf{Hx} = \mathbf{b} \qquad (5)$$

Where

$$\mathbf{Q} = \begin{pmatrix} 2 & -1 & 0 & 0 & \cdots & 0 \\ -1 & 2 & -1 & 0 & \cdots & 0 \\ 0 & -1 & 2 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & \cdots & 0 & -1 & 2 & -1 \\ 0 & \cdots & 0 & 0 & -1 & 2 \end{pmatrix} \qquad (6)$$

and $\|.\|_2^2$ is the L - 2 norm.

The solution of this quadratic optimization problem in equation (5) is obtained using the Lagrange multipliers (Nocedal et al. 2006, Chapter 16) and it is equal to:

$$\hat{\mathbf{x}} = (\mathbf{Q}^T\mathbf{Q})^{-1}\mathbf{H}^T\left(\mathbf{H}(\mathbf{Q}^T\mathbf{Q})^{-1}\mathbf{H}^T\right)^{-1}\mathbf{b} \qquad (7)$$

The taper obtained in equation (7) is named the Block-taper or simply *B-taper*. This taper is guaranteed to be symmetric and therefore has a linear phase. This property is important when the taper is used as pre-Fourier transformation taper to avoid any phase distortion. The proof of this property stems from the following three facts.

1. One can observe that if $\mathbf{x} = (x_1\ x_2 \ldots x_N)^T$ is a solution to equation (2), then its reverse $\tilde{\mathbf{x}} = (x_N\ x_{N-1} \ldots x_1)^T$ is also a solution to the same equation due to the symmetry in the construction of the periodicity (i.e., the axis could be reversed and periodicity still holds).
2. The second order difference of $\mathbf{x}$ is a reversed sequence of the second order difference of $\tilde{\mathbf{x}}$ due to the symmetry of the 3-point operator used in computing second order difference as defined by $\mathbf{Qx}$. Moreover, the L-2 norm of a sequence and its reverse are equal, hence $\|\mathbf{Qx}\|_2^2 = \|\mathbf{Q}\tilde{\mathbf{x}}\|_2^2$.
3. Since the solution $\hat{\mathbf{x}}$ defined in equation (7) and its reverse $\tilde{\hat{\mathbf{x}}}$ have the same cost value and both satisfy the same constraint of the optimization problem in equation (5) and since the solution is unique, then $\hat{\mathbf{x}} = \tilde{\hat{\mathbf{x}}}$ and this proves the symmetry of the taper.

Extension *to higher dimension taper*

Extension to design higher-dimensional tapers follows the same steps as for 1-d case. One can show that for a D-dimensional taper that moves respectively with $(M_1\ M_2 \ldots M_D)$ samples in each dimension, the aggregate sum of the taper's weights is periodic in the i-dimension with a period equal to M_i. A linear system similar to the one in equation (3) can be formulated. The cost function can be constructed as a weighted sum of $L2$-norms of the second order difference of the taper in each dimension. The weights can account for possible variation of the sampling units in each dimension, but also for possible overlap variation. The sum of weighted L-2 norms has a quadratic form and the design problem simplifies to solving a quadratic optimization with equality constraints similar to one in equation (5).

Examples

In the first example, a comparison between a linear-taper and a B-taper is presented. The focus will be on the spectral properties of each taper, particularly the energy level of the side lobes. This is important for Fourier-based data filtering and interpolation processes that aim to extract large amplitudes from the spectrum of the input block data. Having large side lobes potentially may mask some of the weak events. Figure (1-a) shows a liner-taper of length 100 samples and an overlap of 50 samples. Figure (1-b) shows the designed B-taper with the same length and overlap. One can see that both tapers have roughly equal main lob width (Figure 1-c), but the B-taper has lower amplitudes for the side lobes compared to the linear-taper. The B-taper looks very similar to a cosine-taper, however contrary to a cosine-taper it sums to unity when it is moved and merged over the processing window.

A designed two dimensional B-taper with 50% overlap in both dimensions is shown in Figure (2-b). It is compared with a two dimensional linear taper (Figure (2-a)) obtained by multiplying two1-d linear tapers. Their wavenumber spectra are shown in Figure (2-c-d). As for the case of 1-d, the B-taper has weaker side lobes compared to the linear taper. Also because the design is done truly in 2-d, the shape of its main lobe (Figure (2-d)) is radial, offering a better directional spectral properties. The pseudo 2-d linear taper (Figure (2-c)), has a rectangular shape for its main lobe.

Conclusions

Taper design for block processing is often considered as an auxiliary issue. The dimensionality and the spectral properties of the taper should be considered, particularly if there is a need to use it as pre-Fourier transformation taper. The proposed B-taper offers a good choice for both block merging and pre-transformation taper as it is designed to have relatively weaker side lobes compared to a linear taper and a directional spectral property for high dimensions.

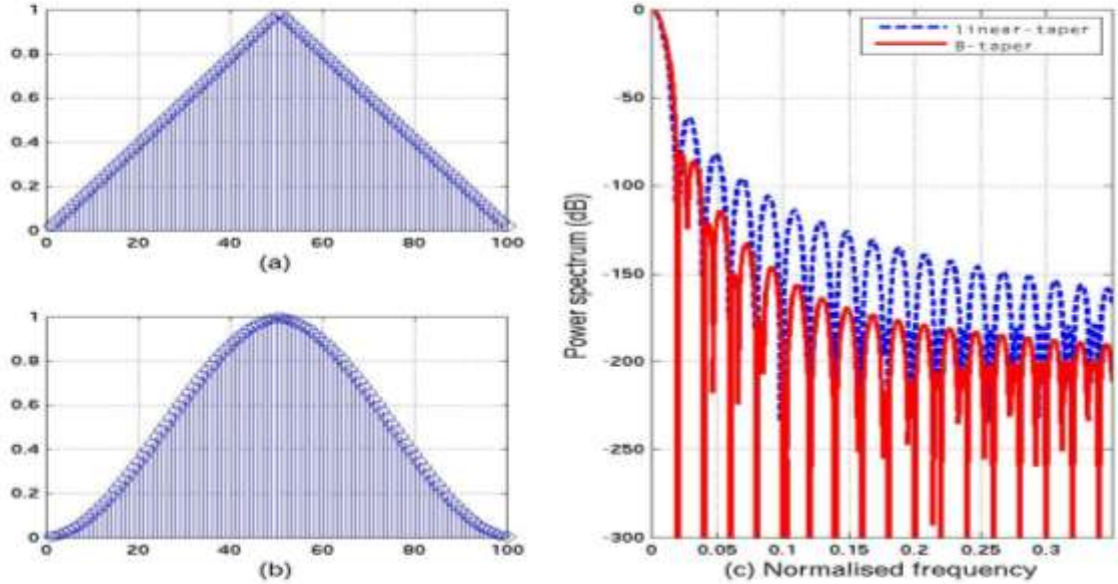

Figure 1 *Comparison of one dimensional linear-taper (a) and a proposed B-taper (b) with length = 100 samples and overlap of 50 samples. Frequency spectra of each taper are shown on the right(c)*

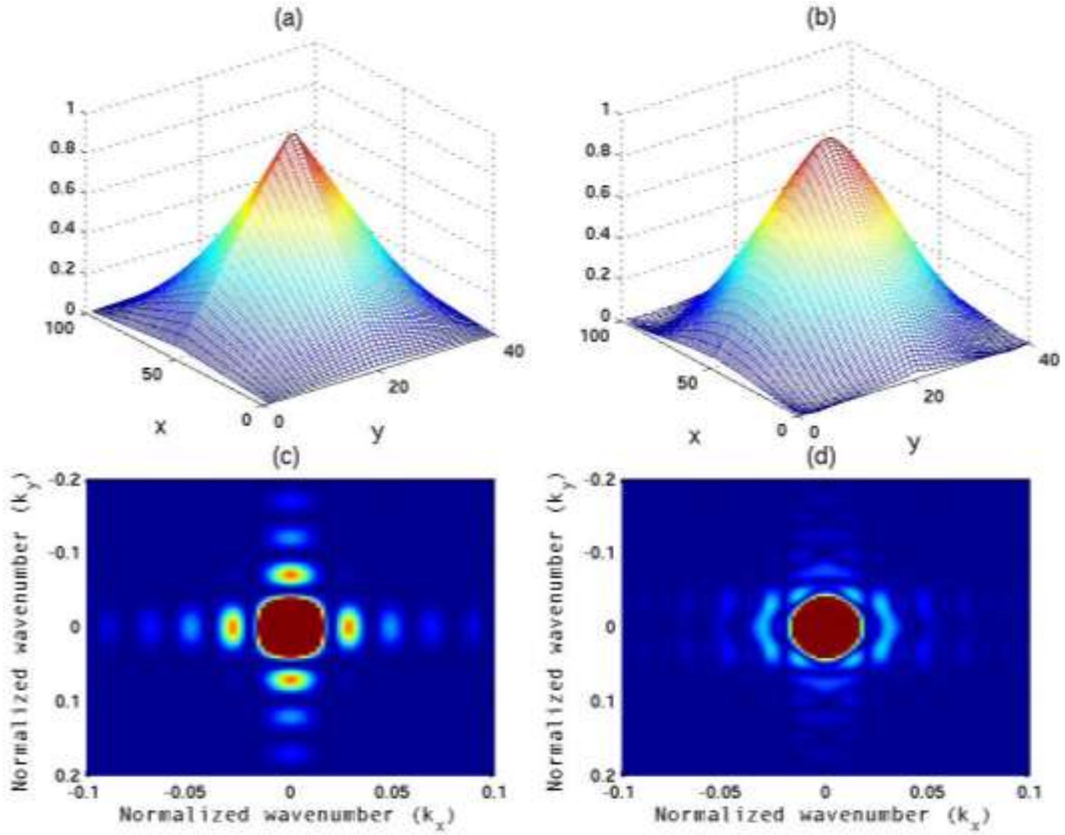

Figure 2 *Comparison of two dimensional linear-taper (a) and a proposed B-taper (b) with size 100 x 40 samples and an overlap of 50% in each direction. Spectra of linear-taper (c) and B-taper (d)*

References

Canales L. (1984), Random noise attenuation. 54th Annual international meeting, SEG expanded abstract

Nocedal G. and Wright S. (2006) Numerical Optimization, Springer Series in Operations Research and Financial Engineering