# Improve Kirchhoff depth imaging using full wave equation traveltimes

*Yongzhong Wang*, Yang He, Alex Yeh, Faqi Liu, Bin Wang and Carlos Calderón, TGS*

**Summary**

Full waveform inversion (FWI) has become the main workhorse to derive highly accurate velocity models and is applied in nearly every depth imaging project. Prestack Kirchhoff migration remains a viable algorithm for validating the fidelity of velocity models derived by FWI because of its flexibility and efficiency in generating Surface-Offset-binned common imaging Gathers (SOGs). However, traditional ray-based traveltime computation becomes less effective as it requires a smooth model. In this paper, we compute traveltime-tables for Kirchhoff depth migration using a full wave equation (acoustic, visco-acoustic, elastic, visco-elastic, etc.), which can directly use high-resolution velocity models without smoothing like those derived from high-frequency reflection-FWI. We demonstrate with both synthetic and field data the benefits brought to Kirchhoff migration by wave equation traveltimes.

**Introduction**

Kirchhoff migration has been an efficient and effective imaging tool for PSDM projects in the seismic data processing industry for decades. However, the traditional ray-based traveltime calculation is usually inaccurate or even fails due to caustics when a velocity model is complex and/or has sharp boundaries such as sediment/salt interfaces. A compromise approach to keep using raytracing is to smooth the velocity model. However, model smoothing may cause the loss of model details which results in inaccuracy of the traveltimes for high resolution imaging. Therefore, using a wave equation solver to calculate traveltimes becomes of interest. Ehinger *et al.* (1996) used a frequency-domain common-offset one-way wave equation to calculate traveltimes. Shin *et al*. (2003) used a frequency-domain common-shot one-way wave equation to calculate the most energetic traveltimes and amplitudes for Kirchhoff migration. Etgen (2012) recapped the derivation of Ehinger *et al*. (1996) and described its applications to different 3D acquisition geometries including streamer and OBC. Their work emphasizes the importance of using wave equation methods for calculating traveltimes in generating SOGs efficiently and effectively. Jin and Etgen (2020) further evaluated the wave-equation-based maximum-amplitude traveltime Kirchhoff migration and applied it to the 3D Thunder-Horse Gulf of Mexico data and noted its benefits for subsalt imaging with SOGs. They pointed out that, for Kirchhoff migration, a low frequency (up to 8Hz) Finite-Difference (FD) wave propagation is sufficient to compute the traveltime-tables to migrate input data up to 50Hz or even higher. Pu *et al*. (2021) extended this approach to use both traveltimes and amplitudes from the wave equation measured maximum-amplitude arrival.

In this paper, we describe and implement a full Wave-equation-based maximum-amplitude traveltime Kirchhoff Depth Migration (Wave-KDM) method and study its benefits compared to a conventional Ray-based Kirchhoff Depth Migration (Ray-KDM). First, we confirm that only a low frequency band (up to 10Hz) FD wavefield propagation is needed. Due to the page-limit of this abstract, we are not going to show the confirmation result here, but in the presentation; second, we test our method on the impulse responses in a 2D BP 2004 salt model; third, we apply Wave-KDM to the 2D Sigsbee2A synthetic data set; fourth, we apply our method to a 3D NAZ field data set having complex geology with shallow salt from offshore Brazil and also to another 3D WAZ field data set from the Gulf of Mexico. Finally, we draw the conclusions for this paper.

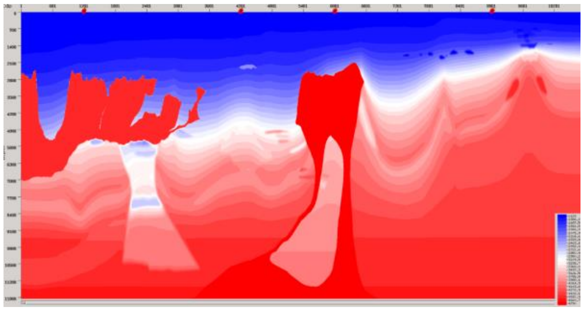
**Method**

By extracting the propagation time from a wavefront produced using the full wave equation, accurate traveltime-tables can be built which can be used in Kirchhoff depth migration. At each location where a traveltime-table is computed, the algorithm involves: 1) propagating the wavefield using a full wave equation (acoustic, visco-acoustic, elastic, visco-elastic, etc.) with a high-resolution velocity model; 2) extracting the traveltime from the wavefront having the maximum-amplitude at each propagation grid point; and 3) interpolating the traveltime to a user specified 3D output cube. By not requiring a smoothed model, the full wave-equation-based traveltime computation is superior to the ray-based method. The compatibility with a high-resolution model inverted by FWI, which is also full wave-equation-based, is a valuable QC. In practice, it can also be a much more efficient and effective QC compared to RTM, because only a low frequency wave propagation is needed for picking accurate traveltimes. The full wave equation generated traveltime-tables can be directly integrated into an existing Kirchhoff migration workflow.

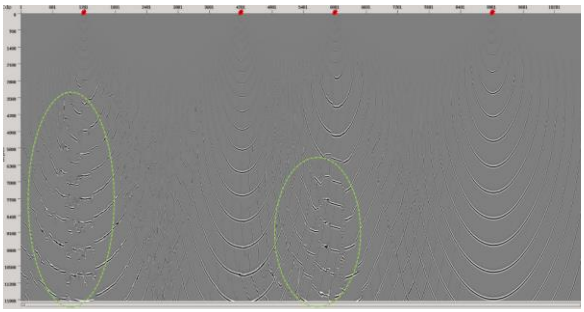
**Numerical tests and applications**

*1) Impulse response comparisons on the BP 2004 model*

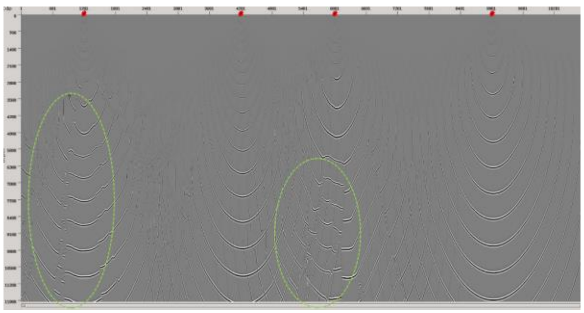
We conducted conventional Ray-KDM and Wave-KDM for zero-offset impulse response tests on the 2D BP 2004 salt model, which has large lateral and vertical velocity variations. Figure 1(a) displays the velocity model, where four sources are injected at the locations CDP=1201, 4201, 6001, and 9001. Figure 1(b) shows the impulse responses obtained by conventional Ray-KDM, whereas Figure 1(c) shows the results obtained by Wave-KDM. On the one hand, we see (highlighted by the dashed green ellipses) that the impulse response results at CDP=1201 and CDP=6001, which are located directly above the complex salt-bodies, are heavily distorted by the Ray-KDM method in Figure 1(b), while the results in Figure 1(c) with the Wave-KDM method are much more coherent and continuous. On the other hand, the results of Figures 1(b) and 1(c) at CDP=4201 and CDP=9001, which are located above relatively simple sediments, are very comparable to each other as expected.



(a)



(b)

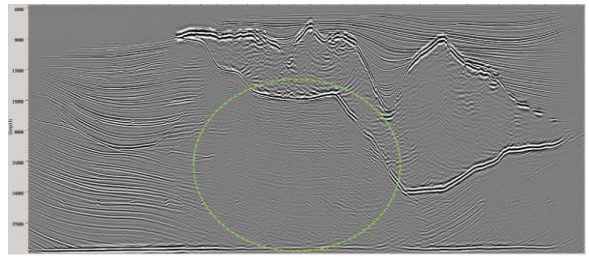


(c)

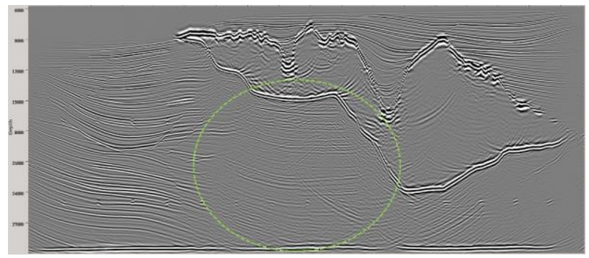
Figure 1: (a) 2D BP 2004 salt velocity model, and impulse response results obtained by (b) conventional Ray-KDM, and (c) Wave-KDM. In the complex subsalt areas (the 1st and 3rd impulse response spread areas highlighted by the dashed green ellipses), Wave-KDM is much superior to Ray-KDM (Model courtesy of BP).

*2) 2D synthetic data example*

We applied Wave-KDM to the 2D synthetic Sigsbee2A data and compared the result to that produced using conventional Ray-KDM. Figure 2 shows the stack images obtained by (a) conventional Ray-KDM and (b) Wave-KDM. We can clearly see that the image from Wave-KDM is of better quality than that from Ray-KDM at the base of salt and subsalt reflectors (highlighted by the dashed green circles).



(a)



(b)

Figure 2: Sigsbee2A model results obtained by (a) conventional Ray-KDM, and (b) Wave-KDM. The base of salt and subsalt events (highlighted by the dashed green circles) are clearly imaged by Wave-KDM in (b) (Data courtesy of SMAART JV project).

*3) 3D field data examples*

***NAZ data example***: We applied Wave-KDM to 3D NAZ field data from offshore Brazil and compared the result to that produced using conventional Ray-KDM. Figures 3(a) and 3(b) show the stack images obtained by Ray-KDM and Wave-KDM, respectively. Obviously, we can see that the salt flank, the mini-basin in between salt-overhangs, and the subsalt events, indicated by the red arrows in Figure 3(b), are better imaged using the Wave-KDM method in terms of amplitude, coherency, and continuity of events. Figures 3(c) and 3(d) compare the corresponding Common Image Gathers (CIGs) at the same locations indicated by the six vertical red lines in Figures 3(a) and 3(b). As indicated by the blue arrows in Figure 3(d), the gathers produced by Wave-KDM are much better imaged than those obtained by conventional Ray-KDM.

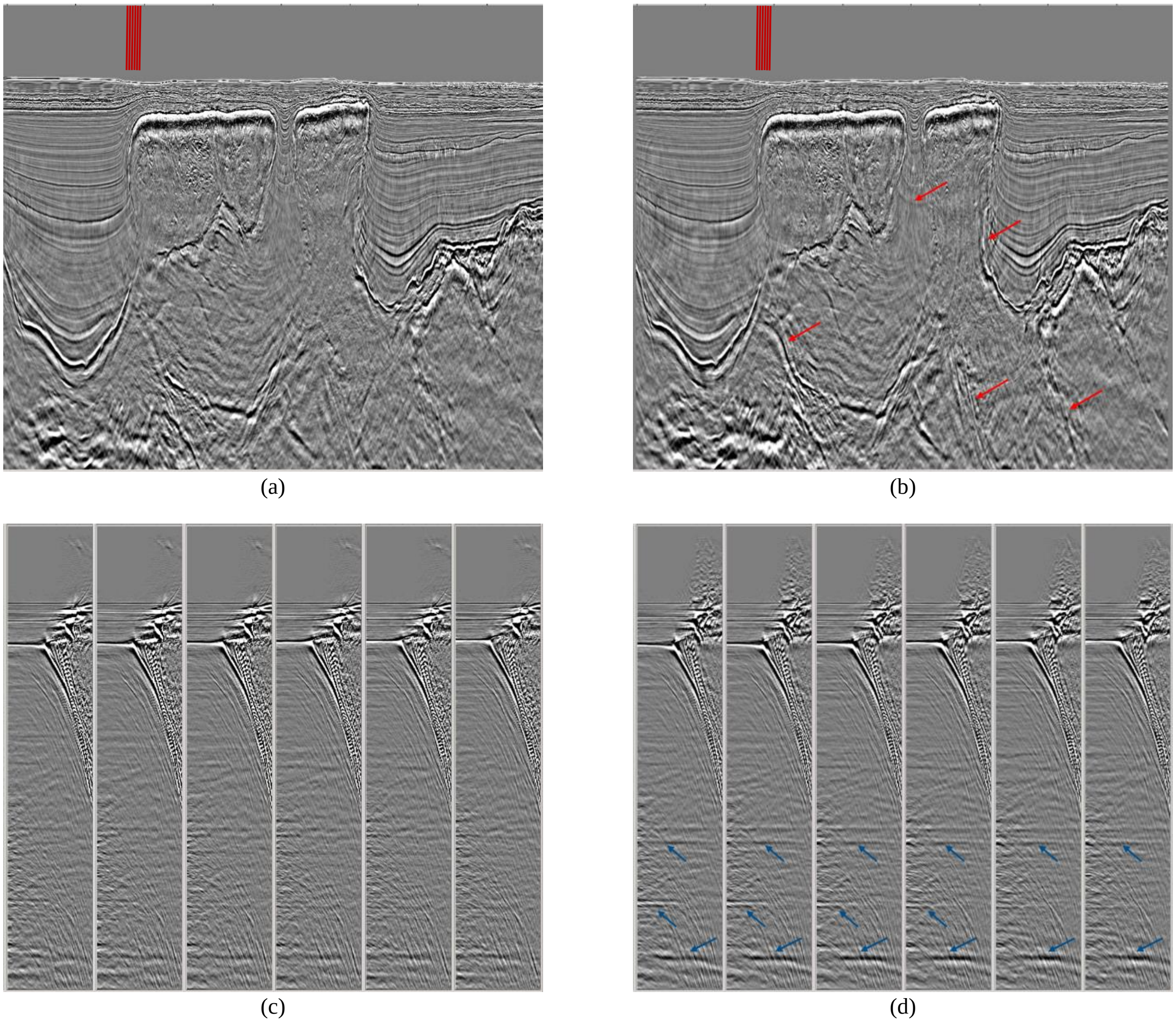



Figure 3: Offshore Brazil NAZ data stack images obtained by (a) conventional Ray-KDM and (b) Wave-KDM. Salt flank, mini-basin, and subsalt events indicated by the red arrows in (b) are much better imaged by Wave-KDM. The CIGs obtained by (c) conventional Ray-KDM and (d) Wave-KDM. From the CIGs, the subsalt events indicated by the blue arrows are greatly improved by Wave-KDM in (d).

***WAZ data example***: We also applied Wave-KDM to another 3D WAZ field data from the Gulf of Mexico and compared the result to that obtained by conventional Ray-KDM. Figures 4(a) and 4(b) are the stack images obtained by Ray-KDM and Wave-KDM, respectively. We can see that the subsalt events, marked with the red ellipse and arrow in Figure 4(b), are greatly uplifted by Wave-KDM. Further, Figures 4(c) and 4(d) compare the five CIGs around the location indicated by the vertical green lines in Figures 4(a) and 4(b). As indicated by the blue arrows in Figure 4(d), we can see that the Wave-KDM gathers for the base of salt and subsalt events are much better imaged than those of Ray-KDM.

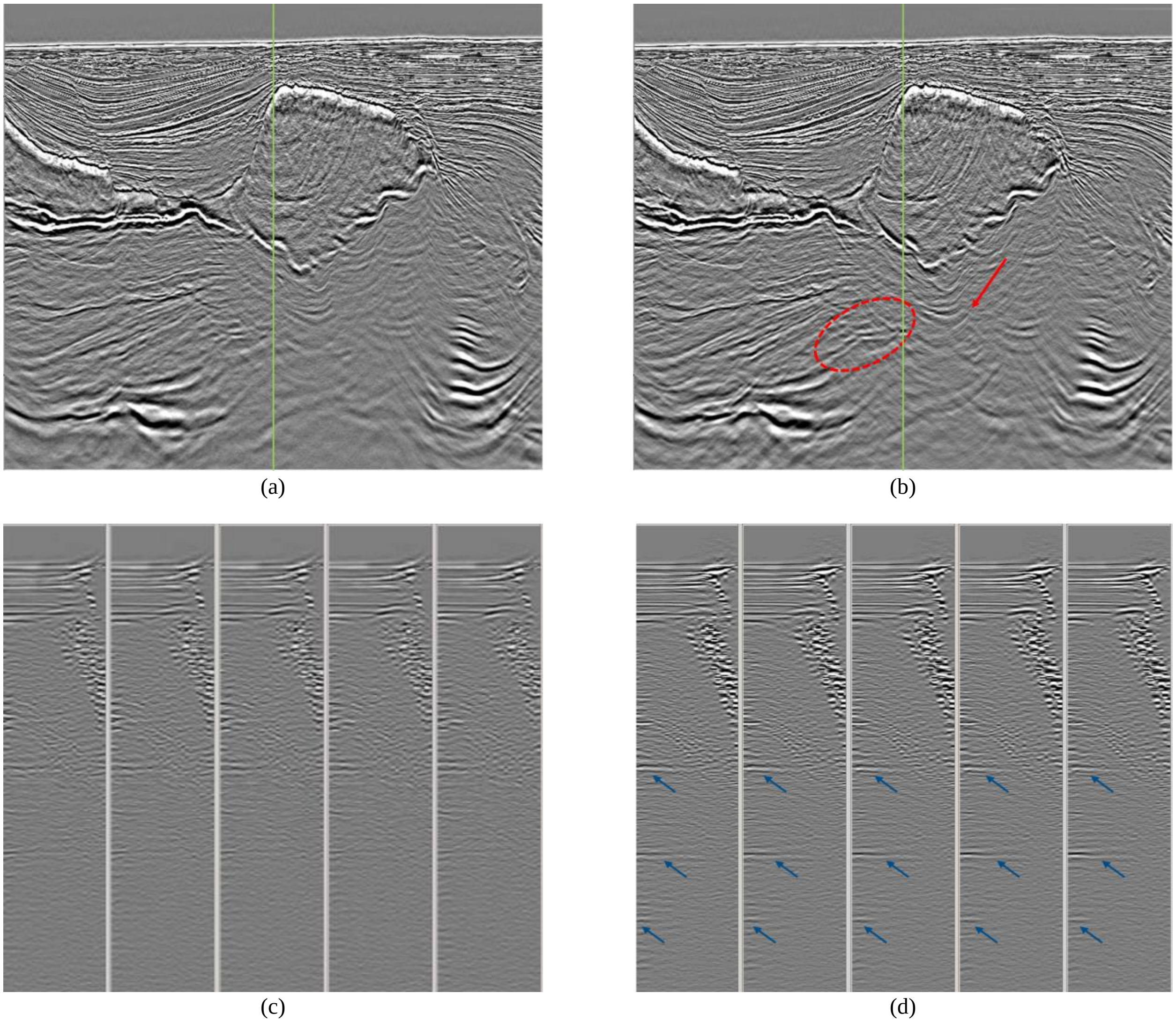



Figure 4: Gulf of Mexico WAZ data stack images obtained by (a) conventional Ray-KDM and (b) Wave-KDM. Subsalt events, highlighted by the red ellipse and arrow in (b), are greatly uplifted by Wave-KDM. The five CIGs around the location indicated by the vertical green lines in (a) and (b) show the base of salt and subsalt events considerably improved by Wave-KDM as indicated by the blue arrows in (d).

## Conclusions

The wave-equation-based traveltime Kirchhoff depth migration is superior to the conventional ray-based Kirchhoff depth migration in complex areas with large velocity-variations because of the more accurate traveltimes obtained by this approach. By not requiring smoothing of high-resolution model inverted by FWI, which is also wave-equation-based, it becomes a more compatible and valuable QC tool.

It is also an efficient QC tool because only a relatively low-frequency (up to 10 Hz) finite-difference wave propagation is needed to produce a full band Kirchhoff image with a quality that is close to that of RTM.

Finally, we would like to point out that a Q-compensated wave equation can be employed to calculate the traveltimes for a Q-Kirchhoff migration.

## Acknowledgments

We would like to thank TGS for permission to publish this work and TGS Multi-Client for permission to use and publish the offshore Brazil and Gulf of Mexico data examples.

**Improve Kirchhoff depth imaging using full wave equation travel time**