

# FWI evolution — From a monolith to a toolkit

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## Abstract

Significant increases in computer power mean it is now possible to routinely apply full-waveform inversion (FWI) techniques to, in principle, produce high-resolution images of subsurface properties. The result delivered by FWI has a different value depending on the geologic situation. In some areas, the FWI-derived earth model may produce a significant uplift in structural image quality relative to that from ray-based tomographic model-building techniques. In other areas, the uplift in structural image quality with FWI may be slight. In these situations, the value from the high-resolution FWI earth model may be obtained during the application of this model in a reservoir characterization workflow. To extract the optimum high-resolution earth model using FWI, in all geologic situations, requires the application of one or more variants on the basic FWI technique — allowing the method to work on both refractions and reflections, with a poor starting model, and in areas with boundaries with high velocity contrasts. This is analogous to many of the other tasks undertaken when processing and imaging seismic data (e.g., demultiple, denoise, and migration) where a single methodology is not adequate to produce the optimum result on all data sets.

## Introduction

Despite many years of incremental improvements, seismic imaging of the subsurface has not yet achieved its full potential. However, the increased cost effectiveness of compute infrastructure is facilitating radical changes to the way that we can undertake seismic processing and imaging projects. In particular, it is now possible to routinely apply full-waveform inversion (FWI) techniques to, in principle, produce high-resolution images of subsurface properties (Tarantola, 1984). However, as we gain more experience in the application of such techniques in a wide variety of geologic regimes and with different data acquisition scenarios, it has become clear that:

- The result delivered by FWI has a different value depending on the geologic situation. For instance, in some areas the FWI-derived earth model may produce a significant uplift in structural image quality relative to that from ray-based tomographic model-building techniques. In other areas, however, the uplift in structural image quality with FWI may be slight. In these situations, the value from the high-resolution FWI earth model may be obtained during the application of this model in a reservoir characterization workflow.
- To extract the optimum high-resolution earth model using FWI, in all geologic situations, requires the application of one or more variants on the basic FWI technique — allowing the method to work on both refractions and reflections, with a poor starting model, and in areas with boundaries with high velocity contrasts. This is analogous to many of the other tasks

that are undertaken when processing and imaging seismic data (e.g., demultiple, denoise, and migration) where a single methodology is not adequate to produce the optimum result on all data sets.

In this paper, we will discuss the contents of such a suite of FWI methodologies and show examples of how they help deliver on the long-held promise that the move to a full wave-based solution to model building and reservoir characterization will lead to significant gains in value from seismic data.

## The challenges of FWI

“...all the current approaches to so-called full-waveform inversion are: (1) always using the wrong data, (2) always using the wrong algorithms, and (3) all too often, using the wrong earth model, as well.”

— Arthur Weglein, *The Leading Edge*, October 2013

The typical application of FWI to exploration seismic data is undertaken, as is so often the way in any branch of physics, using a series of approximations. As the Weglein quote clearly notes, in FWI these approximations typically amount to nonideal data sets (P-wave only measured in a very limited set of offsets and azimuths), nonideal algorithms (the cost of using a fully anisotropic solution to the elastic wave equation is prohibitive in almost all industrial settings), and nonideal earth models (even if an acoustic anisotropic wave propagation is used in forward modeling, it is common to only update the velocity component of the model or, at most, one or two anisotropic parameters). While the alternative direct methods espoused by Weglein (2013) remain beyond the scope of day-to-day industrial use, the question arises: despite acknowledging these limitations, can FWI approaches bring, on a regular basis, useful value to a seismic imaging and processing project? We believe the answer to the question is affirmative; however, a careful, staged approach must be taken to the way the data are used within the inversion process, and, to achieve an optimum result, the user must be prepared to use one or more variants of FWI on any given data set. Finally, depending on the local geologic regime, the value of the FWI result may be more evident during a reservoir characterization workflow (Jones et al., 2018) as opposed to simply providing a step change in seismic image quality.

Perhaps the biggest challenge to a successful application of FWI is the inherent nonlinearity of the inversion process itself. It can be shown that the best way to reduce an FWI to a tractable convex solution is to utilize a multistage approach starting from ultra-low frequencies. Typical suggestions would be as low as 0.5–1 Hz — frequencies that are impractical to record with adequate signal-to-noise ratios in most exploration situations.

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Such ultra-low frequencies not only contain the layer property information required to build an accurate earth model but by their long wavelength nature are not susceptible to the bane of the FWI process — cycle skipping and the associated local minima in the objective function. In the common absence of such ultra-low frequencies, a successful FWI must employ a careful strategic approach — only inverting with that part of the data set which suffers the least cycle skipping possible — at each iteration. Such a strategy involves a judicious choice of shots, frequencies, offsets, and time windows at each iteration to ensure that local minima in the objective function are avoided and that a convergence occurs within an economic time frame.

### From a monolith to a toolkit

It may be observed from any review of the development of any component of a typical seismic processing and imaging workflow that, as time moves on, the technology used tends to progress from a monolithic single solution to a full suite of different solutions. This is a natural result of applying any particular technology to a wide variety of acquisition and geologic scenarios. To get the best result requires the technology to be tuned or modified for a particular situation.

In the case of FWI, the technology has evolved from a least-squares approach that was naturally suited to transmitted waves (diving-wave refractions in a typical surface seismic exploration setting) (see Brittan et al., 2013, for a set of references concerning the evolution of FWI and Virieux and Operto, 2009, for a detailed technical background). Because there has been a desire to use FWI to update the earth model at depths greater than those covered by refracted energy with typical exploration offsets, in recent years the methodology has been extended to explicit reflection FWI solutions (Irabor and Warner, 2016; Vigh et al., 2016; Chazalnoel et al., 2017). Furthermore, efforts have been made (and continue) to alleviate the earlier-described problem of cycle skipping — without the need to record ultra-low frequencies (Jones, 2019, and references therein). Finally, the general nonlinearity of the FWI solution as a whole has encouraged the development of methods that “loosen” the physics used in the wave propagation step at various points in the inversion. These techniques, generally referred to as “extended domain” or “extended parameter” FWI, allow the use of all parts of the wavefield and have the potential to improve convergence toward the correct solution (van Leeuwen and Herrmann, 2013; Wang et al., 2017).

We will briefly summarize the features of typical examples of such FWI solutions and compare the advantages and disadvantages of each approach.

### Refraction traveltimes/phase only FWI

Features:

- Solves the acoustic wave equation
- Optimizes over the model parameters (typically velocity) to minimize the traveltimes errors for a picked horizon in the recorded data
- These methods are distinct from ray-based tomographic solutions in that they use a wave-based propagator for both forward modeling and the adjoint solution (Wang et al., 2018).

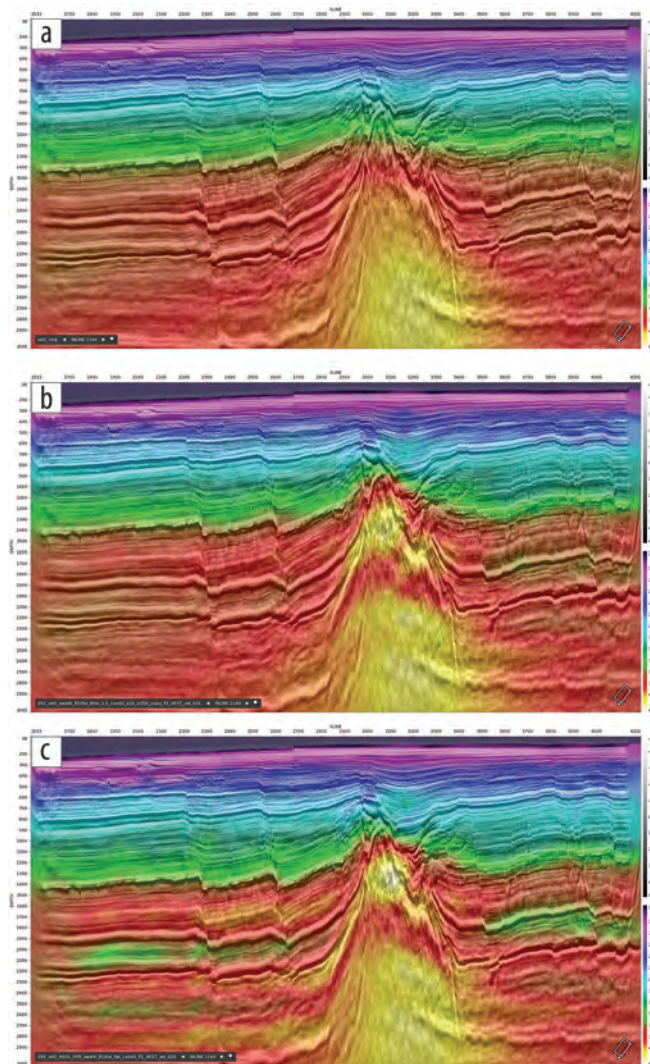
Advantages:

- By removing the shape of the waveform, such an approach helps mitigate cycle-skipping issues and thus relaxes the requirement for good starting models and low-frequency data.

Disadvantages:

- These approaches have a depth penetration limited by the available offset range as they are generally only applicable to the refracted energy. (However, later we describe a version that may be applied to reflected arrivals.)

Figure 1a shows the starting velocity model from the Thoar data set, which is located offshore on the Northwest Shelf of Australia. The starting model for this data set was very simple (essentially a velocity gradient hung off the water bottom). Comparing Figures 2a and 2b, it can be seen that least-squares FWI (LSFWI) using this simple model as a starting model is



**Figure 1.** (a) Initial velocity model for a typical line in a 3D data set offshore on the Northwest Shelf of Australia. (b) The velocity model for this line after 31 iterations of refraction traveltimes FWI. (c) The velocity model for this line after refraction traveltimes FWI and LSFWI. The underlying seismic image is migrated using the initial model and does not change from image to image.

unlikely to converge because the differences between the modeled data and the field data are commonly greater than one wavelet cycle. Hence, it was decided to apply refraction traveltime FWI as an initial inversion. The results of this can be seen in the updated velocity model (Figure 1b) and a better match between modeled and field data (Figures 2a and 2c). After the refraction traveltime FWI was completed, the data match was such that cycle skipping was not considered a significant issue, hence LSFWI was subsequently applied.

### Least-squares FWI

Features:

- Solves the acoustic wave equation
- Optimizes over velocity or anisotropic parameters to minimize the data misfit (Brittan et al., 2013)

Advantages:

- Requires little data preprocessing

Disadvantages:

- Requires good starting models (velocity, anisotropy, etc.)
- Ideally needs low-frequency data
- Diving-wave refraction LSFWI: depth penetration limited by offset range
- Reflection LSFWI: gradient dominated by migration kernel with high-wavenumber update — very difficult to get suitable low-wavenumber updates

Figure 1c shows the velocity model from the Thoar data set after application of refraction traveltime FWI and a subsequent application of LSFWI. The LSFWI was applied after a mute to isolate the refracted energy and was applied in two frequency bands (up to 5 Hz and then up to 8 Hz). It can be seen that the applications of these two types of FWI produce a velocity model that is geologically conformable and that the modeled data match the field data extremely well (Figures 2a and 2d).

### Reflection traveltime FWI

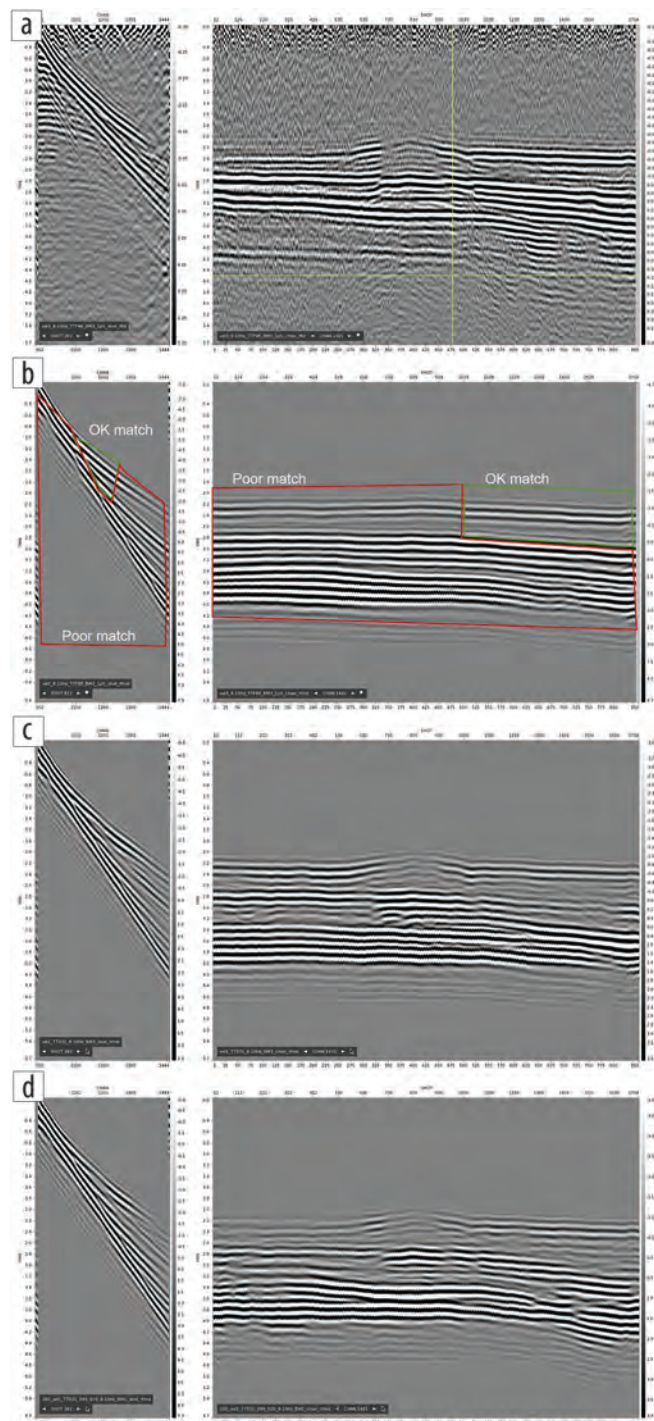
Features:

- Solves the linearized acoustic wave equation (i.e., by single scattering Born modeling)
- Optimizes over model parameters (typically velocity) to minimize the traveltime errors

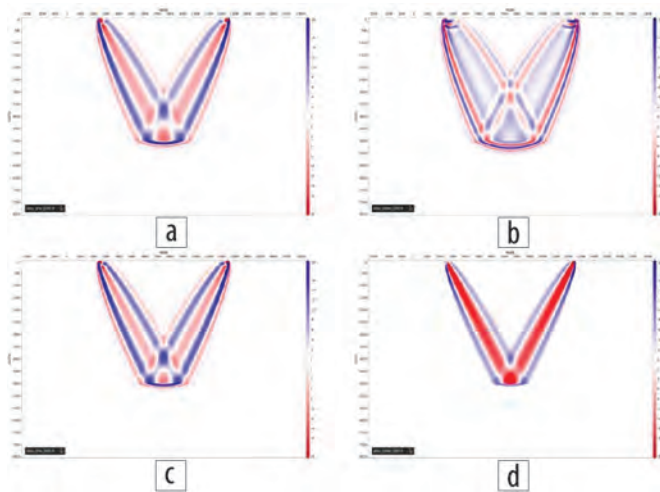
Advantages:

- Increases depth penetration beyond limits of turning waves
- Generates deep low-wavenumber update using reflection energy
- Generates an adjoint source that is less likely to be cycle skipped

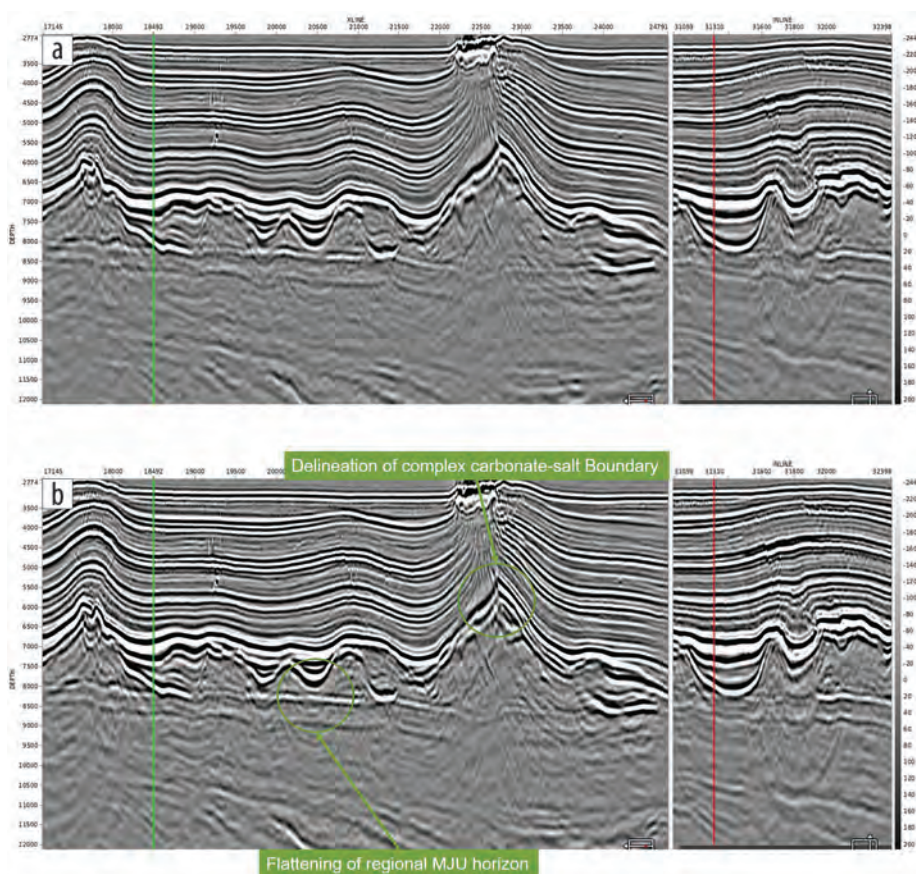
Figure 3 shows examples of a low-wavenumber reflection FWI gradient in the presence of a model that will lead to cycle skipping. In both the case where the model is faster or slower than the true velocity model, the traveltime reflection FWI gradient points in the correct direction of model update, while the LS reflection FWI update struggles to indicate the correct update direction (Wang et al., 2018).



**Figure 2.** (a) An example common shot (left) and common channel (right) gather from the Thoar data set. This field data set has had a high-cut filter applied from 8 to 10 Hz such that it is commensurate with the frequency range used in the FWI process. (b) The same common shot and common channel from (a) modeled using the initial velocity model (Figure 1a). Note that in the vast majority of the data, the phase and amplitude match between the (b) modeled data and (a) field data is poor. (c) The same common shot and common channel from (a) modeled with the velocity model derived using refraction traveltime FWI. It can be seen that the first arrivals now match well across all of the gathers. (d) The same common shot and common channel from (a) modeled with the velocity model derived using both refraction traveltime FWI and LSFWI. The gathers are now very similar to the real data in (a).



**Figure 3.** Comparison of the low-wavenumber FWI gradient for least-squares reflection FWI and traveltime reflection FWI. (a) The reflection FWI gradient for the case where the model velocity is faster than the true velocity. (b) The traveltime reflection FWI gradient for the same case — model velocity is faster than the true velocity. (c) The reflection FWI gradient for the case where the model velocity is slower than the true velocity. (d) The traveltime reflection FWI gradient for the same case — model velocity is slower than the true velocity. Blue indicates a slowdown in velocities in the gradient. Red indicates a speedup. Courtesy of Chao Wang, ION.



**Figure 4.** Example inline (left) and crossline (right) from the Sayeb data set in the Mexican Gulf of Mexico. The green line on the inline marks the position of the illustrated crossline, and the red line on the inline marks the position of the illustrated inline. (a) RTM image migrated using the starting velocity model based on ray tomography. (b) RTM image migrated using the velocity model updated using least-squares reflection FWI.

## Reflection FWI

Features:

- Solves the linearized acoustic wave equation (for instance by single scattering Born modeling)
- Optimizes over model parameters (typically velocity) to minimize the data misfit
- The inversion gradient contains only tomographic kernel from primary reflection such that the updates are concentrated in the low wavenumbers (e.g., Chazalnoel et al., 2017).

Advantages:

- Increases depth penetration and generates deep low-wavenumber updates using the reflection energy
- Requires a number of good starting models (background velocity, reflectivity, and anisotropy)

Figures 4 and 5 show an example of reflection FWI being applied to data from the Sayeb data set in the Mexican Gulf of Mexico. The geology in this region is highly complex, with shallow allochthonous salt, velocity inversions in the Eocene, and carbonates capping the deep autochthonous salt — while much of the exploration interest lies in the presalt geology. The depth of these presalt reflectors means that for the limited offset streamer surveys used in this survey, FWI based on refracted data is not applicable. It can be seen from Figures 4

and 5 that reflection FWI provides important structural changes by inserting subtle and highly resolved variations in both the salt and sediment velocities.

## Extended domain/parameter FWI

Features:

- Solves the wave equation in an L2 approximation; a penalty scalar controls how closely the reconstructed wavefield honors the wave equation
- Optimizes over earth model and reconstructed wavefield jointly to minimize the data misfit and wave equation error

Advantages:

- Increases depth penetration and generates deep low-wavenumber update using reflection energy
- With the penalty scalar set to a large value, the gradients can be difficult to interpret physically.

One of the first steps is to break out from the use of acoustic wave equation FWI toward utilizing more of the recorded signal in the data. One way of doing this is to use an extended domain or extended parameter FWI (van Leeuwen and Herrmann, 2013; Wang et al., 2017). The difference between the extended parameter and acoustic FWI is that we include a data adaption step

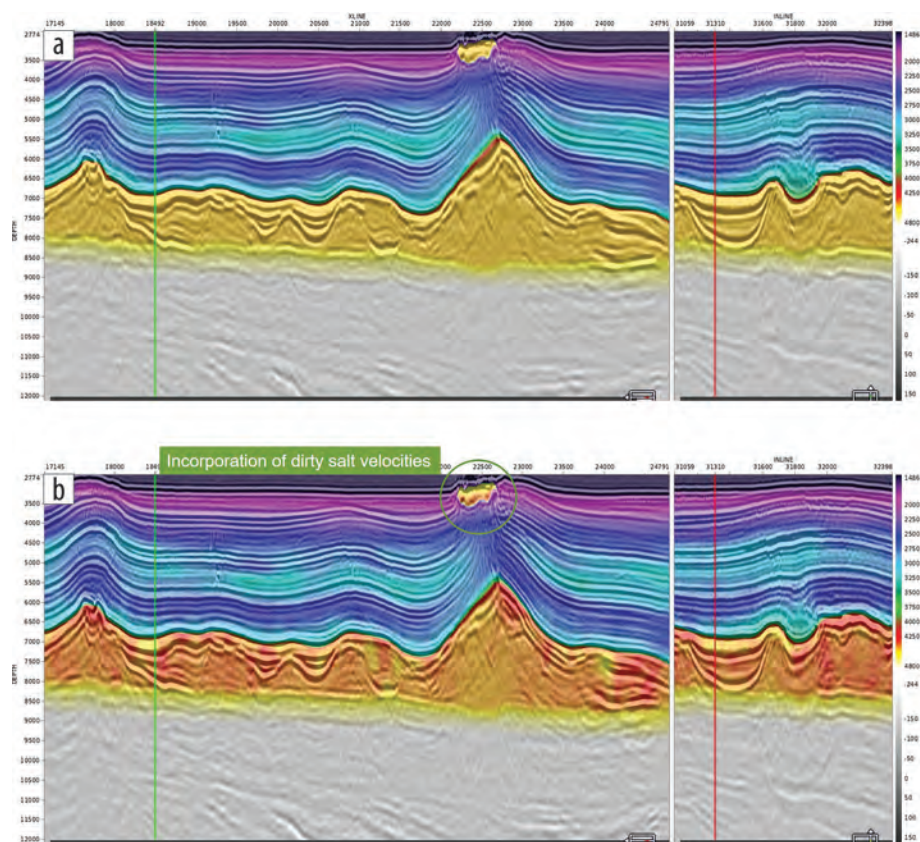
within the inversion. By analogy, in processing we often adapt the data to match the theory used in subsequent processes. For example, we perform multiple suppression so as to prepare the data to meet the assumptions of the subsequent migration algorithm. Contemporary FWI typically uses acoustic, viscoacoustic, or quasi-elastic wave theory, hence the real field data do not match the underlying assumptions of the method. Thus, in extended parameter FWI, we alternate between adapting the data and updating the velocity model.

Figure 6 captures the use of extended parameter FWI to improve the resolution of a velocity model for subsequent use in both imaging and interpretation (Cobo et al., 2018). The starting model for FWI based on ray tomography can be seen in Figure 6a. While this model captures the gross features seen in the coincident sonic log, the resolution is very limited. A subsequent LSFWI using the refracted energy with frequencies from 4 to 11 Hz (Figure 6b) led to a significant increase in velocity model resolution to depths approximately 2 km below the water bottom. This LSFWI model was then used as the starting model for an extended parameter FWI, which led to further refinement of the velocity model at depths down to 6 km (Figure 6c). The structural conformity of this final velocity model can be seen when comparing it with the migrated reflectivity section (Figure 6d); indeed, it has been shown that this final model also offers a considerable uplift when used in a subsequent reservoir characterization workflow (Cobo et al., 2018).

### The road ahead

The evolution of FWI described earlier and the realization that the value of the high-resolution earth model provided by FWI may often only manifest itself within the reservoir characterization process have transformed the application of FWI within exploration and production imaging projects. There remain, however, some key challenges to further successful exploitation of the methodology. First, and most importantly, is cycle skipping. While some of the approaches described (or referenced) earlier can help mitigate cycle skipping, the effect remains a particular hurdle in many applications of FWI, especially in highly complex geologic situations. Significant recent advances in source technology (Brenders et al., 2018) may help by increasing the recorded signal to noise at low frequencies; however, the vast swaths of legacy seismic data acquired with more conventional sources will still suffer from this issue.

In addition, all FWI approaches involve some form of source wavelet. This wavelet may be derived as part of the FWI process itself (Sun et al., 2014); however, in shallow water or for land surveys, as is always a problem in seismic processing, derivation



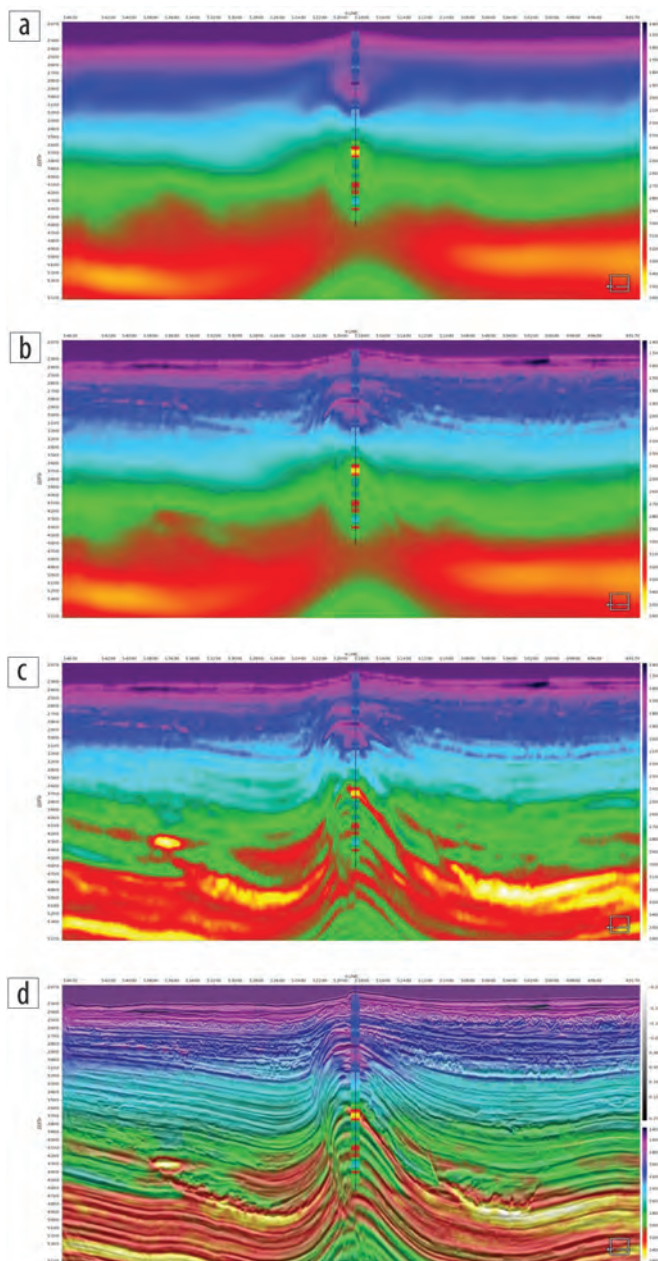
**Figure 5.** The corresponding velocity models overlaid upon the RTM images shown in Figure 4. (a) RTM image and the starting velocity model based on ray tomography. (b) RTM image and the velocity model updated using least-squares reflection FWI.

of an accurate source wavelet may be problematic. While the use of a modeled wavelet is possible, well-known limitations of the physical model used in most common wavelet modeling packages tend to manifest themselves at the low frequencies utilized in FWI. The other significant issue in shallow water can be signal to noise at low frequencies where, particularly for ocean-bottom recordings, there can be significant interferences from surface waves. This issue of signal to noise at low frequencies is also the main hindrance to the widespread adoption of FWI in land imaging projects.

Finally, as we become more confident in the application of FWI and with the array of different wave propagators at our disposal (e.g., vertical transverse isotropy, horizontal transverse isotropy, viscoacoustic, orthorhombic, etc.), it is tempting to extend the modeling parameters that are inverted for in the algorithm. This may have the undesired effect of simply increasing the size of the null space of the inversion (especially if we use parameters that are not sensitive to the data we are trying to match in the inversion) and thus simply giving us more models that fit the data equally well. In these situations, it is sensible to remember the quote from Weglein (2013) and only try and invert for model parameters that the data support. **FILE**

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**Figure 6.** (a) The starting velocity model for an example line from the Anegada-Labay data set in the Mexican Gulf of Mexico. This velocity model was derived using ray tomography. The sonic velocities from a coincident well are superimposed on the plot. (b) The FWI velocity field for the same line as in (a) derived using refraction LSFWI. (c) The final FWI velocity field for the same line as in (a) derived by refraction LSFWI and subsequent extended parameter FWI. (d) The final FWI velocity field for the same line as in (a) with the Kirchhoff migrated structural image overlaid. Note the structural conformity of the velocity field (Cobo et al., 2018).

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### Data and materials availability

Data associated with this research are confidential and cannot be released.

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