# Tutorial: The kinematics of migration. Part II

This two-part tutorial is abstracted from the new EAGE publication: 'An Introduction to Velocity Model Building'

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## Abstract

Migration is the process that builds an image from recorded seismic data, by (ideally) repositioning the recorded data into its 'true' geological position in the subsurface. The propagation of seismic waves can be described by either wave theory or ray theory, and the numerical approximations to working with these descriptions can be implemented in various transform and data domains. Furthermore, there are two main approaches to performing migration: Time Migration, and Depth Migration, both of which can be performed either after stack or before stack.

In Part I of this tutorial, I discussed the concepts involved in migration, highlighting the major differences between time migration and depth migration, to give readers some insight into why depth migration is important in providing a reliable image of the subsurface. The concepts of ray-based and wave-based descriptions of migration were also introduced. In this second part of the tutorial, I'll discus ray-based techniques, algorithm noise, multi-pathing, and one-way versus two-way propagation. Note that the figure numbering continues from Part I, so that reference to them can be readily made.

#### Ray-based (integral or summation) techniques

Integral migration techniques such as Kirchhoff, equivalent-offset, and beam, set-out to solve a representation of the wave equation using a high frequency approximation, whereby each arrival is treated as a spike-like event, and the superposition of these events with appropriate amplitude scaling, reconstructs the final image through superposition of stationary phase components. A fundamental feature of these techniques is that the image can be computed for a subset (e.g., for a gather, a depth slice, an image line, etc.), and the greatest strength of integral techniques (in comparison to wavefield extrapolation techniques) is their cost-effectiveness for producing these subsets of migrated data. The dip limitation also is specified readily in integral techniques during travel time computation or during the summation step. In addition, integral techniques are very well suited to cost-effectively imaging the steepest dips, and can be modified to image turning (diving) rays. It should be noted that for production of just the 3D image (without gathers) a WEM technique can be less computationally intensive that a Kirchhoff scheme, as the cost of a WEM migration is roughly proportional to the number of shots, whilst a Kirchhoff scheme is more or less proportional to the number of input and output traces. Hence for multistreamer data with many traces per shot, WEM can be cost effective if we aren't creating gathers.

The most widely used technique in this category is the single-arrival Kirchhoff integral, which usually is implemented in the time-space domain (but can be implemented in the frequency-wavenumber domain, e.g. Etgen et al., 1997). In Kirchhoff migration, the migration process is separated into two stages: computation of the travel times along ray-paths through the velocity model (figure 14), and subsequent summation of information associated with these travel paths (figure 15). Other techniques include the equivalent-offset scheme introduced by Bancroft (Bancroft and Geiger, 1994) and various beam migrations. The Gaussian beam technique pioneered by Popov in the Russian literature (Popov, 1982 a; Babich and Popov 1989, Popov et al., 2007) and others (Cervény, 1981; Cervény et al., 1982; Hill, 1990, 2001; Sun et al., 2000) is more complicated to implement, but does have the advantages of dealing with multipath arrivals and of keeping costs down by computing operators only in the vicinity of a narrow trajectory (Wang and McClay, 1995). This technique also can be implemented in different domains (e.g., Lazaratos and Harris, 1990). For beam migration, we can conceive of there being three stages in the process: measurement of the time-slopes present in the input data on common shot, receiver, or offset gathers, computation of ray paths and travel times associated with these time slopes, and summation of information associated with these travel paths. The most complete of these schemes is the Gaussian beam technique, but more approximate schemes can also be implemented (Gray, et al., 2009), and go under various names such as fast beam, parsimonious beam, and controlled beam, etc.

For beam schemes, the objective is to link the surface take-off (emergence) angles at both the source and receiver locations to the possible ray paths that impinge on a given subsurface reflector segment (figure 16). This is done for all subsurface segments, and an image computed using only contributions close (within a Fresnel zone) of this ray corridor. To convert the time-slopes to angles, we need a velocity field, and this is updated in an iterative way, as for other migrations. The main advantage of a beam scheme over a Kirchhoff technique is that the slope tables need only be estimated once, so subsequent construction of an image for a different velocity model is very fast. Conversely, for a Kirchhoff scheme, we need to recomputed the travel time tables whenever we modify the velocity model.



Rays are traced in the velocity model from a coarse grid of surface locations to a finer mesh of subsurface image points. BUT the actual shot and receiver locations are not on the surface grid nodes, and the subsurface points are not the image points. Hence we interpolate.



Kirchhoff migration copies energy from the input trace everywhere along the impulse response. However, only a small part of this contributes anything useful to the image: the rest either cancels or produces noise.



Slopes picked on shot or receiver gathers are related to the surface take-off (emergence) angles. The slope in the offset plane is also related.

Once the travel times (for Kirchhoff) or ray angles (for beam) have been computed, we then need to select samples that will contribute to each image point. For Kirchhoff migration, we collect the samples within some aperture and dip limit for which the travel times have been computed. For beam migration, we collect data samples in the vicinity of the computed ray tube (or 'beam'), such that a Fresnel zone is encompassed, and only coherent energy thereby summed to form the image. In some beam schemes, a representative wavelet is used to emulate the data at each contributory picked dip segment, and these wavelet contributions summed to form the image.

For some beam implementations, we pick local time slopes in the input shot and offset domain gathers. Within these gathers, local tau-p measurements are made in a sliding window and combined with thresholding on coherency to select the dominant constituents of the data. The slopes are converted to surface take-off (emergence) angles using the current estimate for the velocity field, and then ray paths are computed from the surface source and receiver positions, and travel times along these paths analysed to determine the intersections of the path from the source and receiver sides, so as to find the image point for this particular ray path. Energy associated with this image element is then summed into the output image space taking account of the Fresnel zone. The parameterization of the slope picking is sensitive: we need to select a window width for the slope fitting in the gather (sometimes referred to as a beam width), and to decide how this window moves across the gather. The range of the slope scan within this window then needs to be determined, bearing in mind that far traces might be aliased, and that we may have several possible slopes at any given point in the gather. We also need to link (i.e. pair-off) the slopes picked for a point in the two domains (as there may be two or more slopes detected at any given point).

# Algorithm noise in integral techniques

All implementations of migration will create some kind of noise in the output image, as they are not perfect solutions of the wave equation, and in addition, the input data sampling may not meet all the requirements of the algorithm being used (e.g. regular spatial input sampling). Mostly this created noise will be insignificant, but for some algorithms it will be worse than for others. For example, a Kirchhoff migration builds an image by copying a sample of input data out along the 3D impulse response curve for the velocity model associated with the corresponding part of the subsurface (figure 15), and the sum of all such responses build the output image. Some of the energy spread along this impulse response will interfere constructively if within the Fresnel zone of the actual reflector (i.e. the principle of stationary phase) to contribute to the output image (figure 15), but the remainder of this energy does not contribute, and the hope is that due to destructive interference, that it will simply cancel out and 'go away'. The degree to which it does cancel depends on the input trace spacing, regularity of this spacing, and high-frequency content.

In practice, some of the non-contributory energy remains in the output image as a form of steeply dipping (sometimes aliased) noise. Substantial protection against this effect is afforded by filtering-out aliased energy from the migration operators prior to summing to form the image (e.g. Gray, 1992; Lumley et al. 1994; Abma et al. 1999). Wavefield extrapolation techniques will leave less noise, and a beam migration (although similar to Kirchhoff in that it uses ray tracing) will also have less noise, as the beam technique only computes a contribution to the output image in the vicinity of the Fresnel zone. In figures 17 and 18 we see a real example comparing a Kirchhoff image and a beam image: the former has a class of dipping noise that tends to make the reflectors look 'choppy' or 'broken-up'. A similar class of noise is created by irregularly sampled input data, and this can be mitigated by interpolating and regularizing the input offset volumes to bin-centres.



Kirchhoff migration with horizons showing a 'choppy' appearance due to algorithm noise



Corresponding beam migration with less noise

#### Multipathing

Multipathing refers to the fact that energy can propagate from the surface to a reflecting element in the subsurface via several possible routes (figure 19). A conventional 'single arrival' Kirchhoff migration scheme computes only one possible ray path associated with the velocity model, hence is restricted in its ability to construct an accurate image in regions where multipathing occurs. This is not an inherent limitation of the Kirchhoff scheme, but rather industrial expediency, as computing more than one arrival branch increases the cost. Conversely, a beam technique will be able to cost-effectively handle a multi-pathing problem, as it is not restricted to one particular travel time path, but rather to whatever events its slope picking phase identifies. This detail is also related to the desirable coupling of migration algorithm and model building scheme. Below a salt body, a single-arrival Kirchhoff migration is inappropriate as it will not capture all the required image energy, and part of the energy not correctly captured will appear in the gathers as a class of noise. Remember that this multipathed energy is present in the input data and looks just like any other event in terms of its moveout behaviour. Migrating such data with a single-arrival migration scheme does not cause this energy to disappear, but rather it appears in the output gathers and image as spurious events and/or noise. Hence using such corrupted gathers as input to a model update scheme will yield an unreliable velocity model: the autopicking of these poorly Migration tutorial: I.F. Jones

behaved gathers will produce bizarre results, and the subsequent inversion will yield novel and unusual values of velocity! Figure 20 shows a Kirchhoff image of a North Sea salt dome, where the image below the overhanging flanks of the dome is poor. Conversely, in the wavefield extrapolation migration (WEM) image in figure 21, we have better definition of the steep sub salt reflectors, as an image built via extrapolation inherently deals with multi-pathed arrivals.

# Figure 19

In complex environments, there can be more than one path from a surface location to a subsurface point



A Kirchhoff scheme usually only computes travel times for one ray path... what happens to the energy from the rest of the ray paths from input data?

Multipathing - there is more than one possible route from the surface to the reflector



Anisotropic 3D Kirchhoff preSDM – fails below the salt body due to multipathing



Anisotropic WEM provides a better image in this case

#### One-way versus two-way wave propagation

One of the steps in obtaining a simplified solution to the wave equation involves approximating a square root term (in the dispersion relationship of the wave equation). As for all square roots, we have two possible solutions: a positive and a negative root. In the context of wave propagation, the physical interpretation of these roots corresponds to energy coming up and energy going down in the earth. A simple rendition of migration (the one-way propagation schemes) will deal only with upcoming energy (energy captured by the receivers) the has not changed its direction of vertical propagation from the source to the reflector or from the reflector to the receiver. This excludes many arrival paths; namely those that have undergone double bounces (Bernitsas et al., 1997, Cavalca & Lailly, 2005) or have undergone continuous refraction (turning or diving rays). Two-way propagation refers to ray paths that change direction either on their way from the shot down to the reflector, or coming back up from the reflector to the receiver (figure 22). Source and receiver ghosts as well as multiples also fall into this category. Almost all of the migration schemes ever used have been one-way schemes, with the exception of turning-ray Kirchhoff or beam migration (turning-rays being a sub-set of two-way propagation). Hence great effort is taken in pre-processing the input seismic data to remove events associated with two-way raypaths prior to (one-way) migration.



## Conventional one-way propagation as assumed by standard migration schemes



No change in propagation direction on the way from the surface down to the reflection point Two-way propagation: requires a more exact solution of the wave equation to migrate such arrivals



The direction of propagation changes either on the way down from the surface to the reflection point, or from the reflection point back up to the surface

Two way travel paths

Recently we have seen renewed interest in the two-way wave equation (McMechan, 1984; Baysal, 1984; Wapenaar, et al., 1987), both with reverse time migration (Whitmore, 1983, Yoon, et al., 2003, Bednar et al., 2003, Farmer, 2006, Zhou, et al., 2006) and other more approximate wavefield extrapolation techniques (Shan & Biondi, 2004, Zhang et al., 2006). Reverse time migration (RTM) properly propagates the wave-field through velocity structures of arbitrary complexity, correctly imaging dips greater than 90 degrees. It even has the potential to image with internal multiples when the boundaries responsible for the multiples are correctly represented in the velocity model.

Standard shot-based one-way wavefield extrapolation (WE) preSDM techniques image the subsurface by continuing (extrapolating) the source and receiver wave-fields for each shot. The imaging condition is invoked by cross correlating these two wave-fields at each depth level, and then summing the contributions from all shots in the aperture to form the image. One of the assumptions made in using this technique is that the wave-fields travel along the direction of extrapolation only in one direction: downwards for the source wave-field, and upwards for the receiver or scattered wave-field. In practice, each of these wave-fields will generally travel both up and down when the velocity model is complex, when turning (diving) ray-paths are involved, or when multiples are being generated. Solving the full (acoustic) two-wave equation, using for example RTM, could in principle image multiples and double bounce arrivals, if we have an accurate enough model to work with (and could deal with boundary conditions adequately).

In figure 23, we see a conventional one-way WEM image from a deep water West African salt province and the corresponding RTM image in figure 24. Both images used the same model. It is clear that the WEM result is missing the steeply dipping salt flank (as it will be illuminated only with turning and/or double bounce arrivals). Also, the WEM has a class of noise in the image which is probably the result of the two-way arrivals present in the input data being mispositioned in the one-way image. In this case, the model was built using ray-based tomography and WEM images to pick the salt geometry. If an approach more in keeping with what RTM can achieve had been used, then the RTM result could have probably been further improved.



3D WEM (one way migration) of complex West African deep water salt body



RTM using same input data and same model

#### Summary

The discussions here together in Parts I and II of this tutorial have set out to familiarize the reader with the basic principles of migration and of the main approximations used to numerically solve the wave equation (Robein, 2003; Biondi, 2006). An understanding of these approximations is useful so as to appreciate the practical limitations each algorithm has, so as to make a more informed decision as to when and where to employ a given technique.

Apart from the above technical aspects related to how algorithms function, there is also a difference in the way we need to work. Historically, when time migration was being used, the oil company interpreters would at best monitor the processing, and wait until the pre-processing was finished, the velocities picked, and the time migration run, before beginning the interpretation process. What was then passed-on to the interpreter was the final product from the view point of the geophysicist. Interpretations of layers from the time migrated volume would be made and later converted to depth using wells for calibration. Thus, the process was purely sequential. Conversely, depth imaging is an iterative multi-disciplinary effort, involving ongoing input from the oil company interpreter during several of perhaps many iterations of model update and (depth) migration. The interpretation may evolve during this process, as understanding of the prospect changes and is refined. Conversion from geophysical depth to geological depth may still need to be made (either on the interpreted depth horizons, or the depth volume), depending on whether we've been able to adequately address anisotropic effects, or localized heterogeneities.

Hence the complexity of the velocity model can evolve not simply because of the model update process being used, but also due to changes in any preconceptions that the interpreters might have, and additionally, their practical geological insight may also rule-out implausible inversion results. Due to the various limiting assumptions of the migration schemes available, it is important to couple the complexity of the algorithm to the complexity of the geological problem, and also to ensure that the velocity model building scheme is based on comparable (compatible) assumptions to the migration scheme.

As a final comment, we need to be aware that different migration algorithms make differing assumptions about the behaviour of the subsurface, and are based on varying mathematical simplifications of the acoustic wave equation. These limiting assumptions may have unacceptable consequences if we are using a given algorithm as part of the model update loop in an imaging project. We need to match the performance of the algorithm we select to the complexity of the subsurface model we expect to build, and image we hope to see.

Table 2: time-line for evolution of industria	l technique	es
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Period in use as	Technique	Common domain & type of application
primary deliverable		
1975- 1988	2D postSTM	Finite Difference (FD) (x,t) & (x,f)
		Initially with 30°, then 45° and later 60 ° dip limits
1980-1988	2D postSDM	FD (x,f)
		Initially 45° and later 60° dip limits
1985-1995	3D postSTM	FD (x,y,f)
		Initially with 45° and later 60° dip limits
1990-2001	DMO + 3D zero-offset constant velocity	Constant velocity phase shift (Stolt) zero offset
	preSTM, followed by a de-migration of the	preSTM, and subsequent de-migration, in
	stack and then 3D postSTM	conjunction with FD (x,y,f) postSTM
1990-1995	2D full-offset preSDM	FD focussing analysis interactive (x,f)
1993-1997	DMO + 3D zero-offset constant velocity	Constant velocity phase shift (Stolt) zero offset
	preSTM, followed by a de-migration of the	preSTM, and subsequent post-stack de-migration,
	stack and then 3D postSDM	in conjunction with FD (x,y,f) postSDM
1995 - present	Full-offset v(x,y,z) 3D preSDM	Kirchhoff (x,y,z) isotropic
2000-2003	Full-offset v(x,y,t) 3D preSTM	Kirchhoff (x,y,t) straight ray
2002-present	Full-offset v(x,y,t) 3D preSTM	Kirchhoff (x,y,t) curved and turning ray &
		anisotropic
2000-present	Full-offset v(x,y,z) 3D preSDM	Isotropic wavefield extrapolation (WE), either with
		for example: FD, SSFPI, & non-WE beam
2000 - present	Full-offset v(x,y,z) 3D preSDM outputting aathers	TTI Kirchhoff (x,y,z) anisotropic turning ray
2005-2008	Full-offset v(x.v.z) 3D preSDM outputting	VTI wavefield extrapolation, either with for
	gathers	example: FD. SSFPI. and alternatively non-WE
		beam
2006- present	Full-offset v(x.v.z) 3D preSDM	VTI two-way wavefield extrapolation using reverse
,		time migration, or two-pass one-way extrapolation
2008- present	Full-offset v(x.v.z) 3D preSDM	VTI beam or two-way wavefield extrapolation
	outputting gathers	using reverse time migration
2009- present	Full-offset v(x,y,z) 3D preSDM	TTI beam or two-way wavefield extrapolation
	outputting gathers	using reverse time migration

(adapted from Jones et al., 2008)

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