Tutorial: The kinematics of migration. Part I

This two-part tutorial is abstracted from the new EAGE publication: 'An Introduction to Velocity Model Building'

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Abstract

Migration is the process that builds an image of the subsurface from recorded seismic data, by (ideally) repositioning this data into its 'true' geological position in the subsurface. To perform migration, we need to understand and to be able to describe, how sound waves propagate in the earth. The propagation of seismic waves can be described by either wave theory or ray theory, and the numerical approximations to working with these descriptions can be implemented in various transform and data domains. Furthermore, there are two main approaches to performing migration: Time Migration, and Depth Migration, both of which can be performed either before stack or after stack.

I'll briefly discuss the concepts involved in migration, and highlight the major differences between time migration and depth migration, so as to give readers some insight into the reasons behind why depth migration is important in providing a reliable image of the subsurface. In contemporary industrial processing, both time and depth migration need an estimate of the subsurface velocity field in order for the migration process to proceed, but for the more demanding process of depth migration, a more accurate velocity model is needed. The discussion here will concentrate on the way different wave-based and ray-based migration schemes handle positioning of events (the kinematics), rather than the associated amplitudes (the dynamics), and will review some of the approximations made and physical consequences of these approximations.

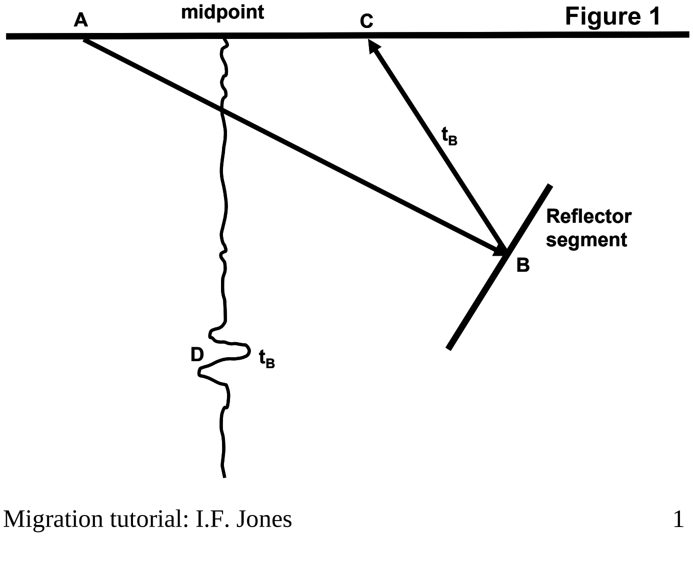
In Part I of this two-part tutorial, I'll address: the objectives of migration, time versus depth imaging, the migration response, waves versus rays, velocity scale-length, domains of application, and the evolution of migration schemes. In Part II I discus ray-based techniques, algorithm noise, multi-pathing, and one-way versus two-way propagation.

What migration sets-out to do

Sound waves that propagate through the earth and reflect off subsurface horizons are the raw data we record at the surface. With ray theory or wave theory we hope to describe the motion of these waves so that we can construct a subsurface image from them. However, full solution of the elastic wave equation is not something that we usually set out to achieve. Instead, from the standpoint of industrial expediency, we make various simplifying assumptions involving a progression of solutions ranging from the simpler to the more complex (Bednar, 2005; Pelissier, et al., 2007). Not surprisingly, this progression has moved in tandem with the increase in available computer power, and development of interactive model update tools for velocity and anisotropy parameter estimation (Jones, et al., 2008).

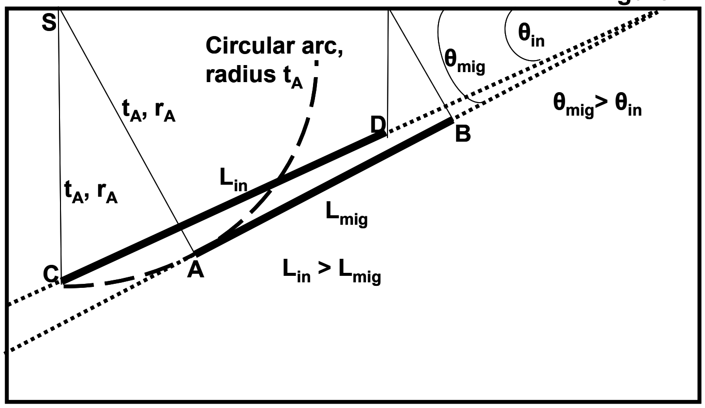
Figure 1 shows the geometry of a reflector and the signal recorded at the surface from an incident sound wave plotted at its CMP location (Chun & Jacewitz, 1981). From Figure 2, it can be noted for the zero-offset raypath shown that during migration, a segment of input recorded surface seismic data (CD) is re-positioned to its (approximate) subsurface position (AB): the process results in a shortening of the segment length (AB<CD) and an increase in the reflector segment dip ($\theta_{mig}>\theta_{in}$) such that $\sin \theta_{mig} = \tan \theta_{in}$, for both time and depth migration. So, to reposition a recorded event, we can think of migration as swinging the element at location C, up through an arc of radius r_A. This process within the migration creates what we call the migration operator. For time migration, the operators in 2D are symmetric circular arcs for zero offset recordings and symmetric elliptical arcs for other offsets. For depth migration, the operators are more complex, as the travel times have to be converted to distances via a spatially variable velocity field (rather than using the simplified locally 1D vertical velocity function assumed by time migration). Many excellent text books deal with these basic principles - see for example, Clarebout, 1976; Yilmaz, 1987; Claerbout, 1992; Bancroft, 1997; Berkhout, 19xx; Fagin, 1999, Robein, 2003, Biondi, 2006.

Figure 1



The two-way raypath from the surface location A to subsurface reflector position B, and then back to surface location C, has total travel time t_B. Location D is located below the midpoint between A and C, and is where the recorded energy sits on our seismic field records after reflecting from true reflector position B.

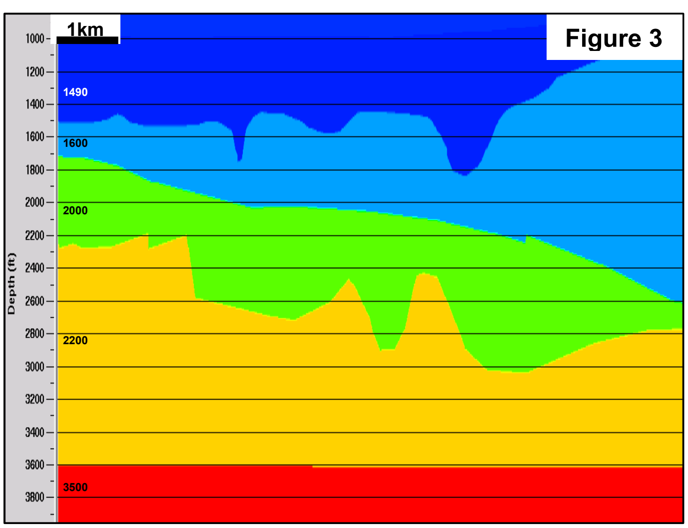
Figure 2



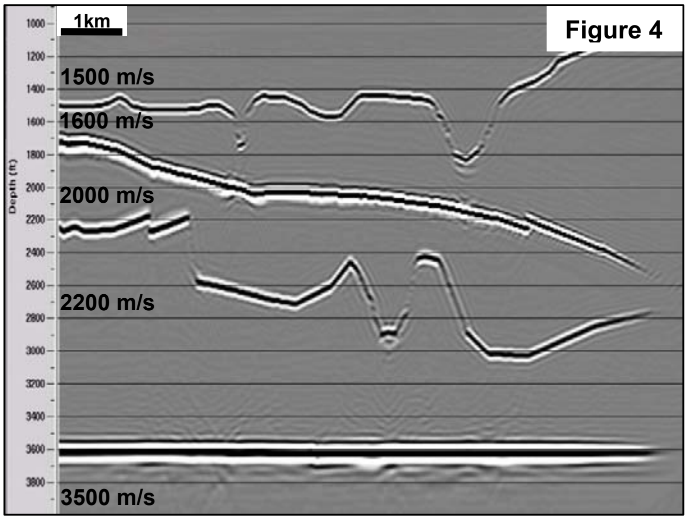
The raypath from the surface location S to subsurface reflector position A is of travel time t_A: this also defines the time to the position C directly below S. Location C is where our recorded energy sits on our seismic field records after reflecting from true position A. For a medium with constant velocity V, we could draw the diagram for length r_A, with $r_A = V.t_A/2$

Time versus depth
Think of the analogous situation of light bending (refracting) as it passes through an interface between two different materials with different refractive indices, say air and water. Sound also bends (changes direction) as it passes through an interface (at some angle away from the normal) between two different materials that have different sound-speed. In seismic data processing, the process of depth migration is designed to compensate for the effects of this refraction (bending of the travel paths) at interfaces or within layers when there is a velocity gradient, so that the image of the subsurface appears in its correct (geological) position. Time migration however, ignores lateral velocity change when computing the underlying migration operator. Hence, at least locally, all refracting reflector segments appear flat to it.

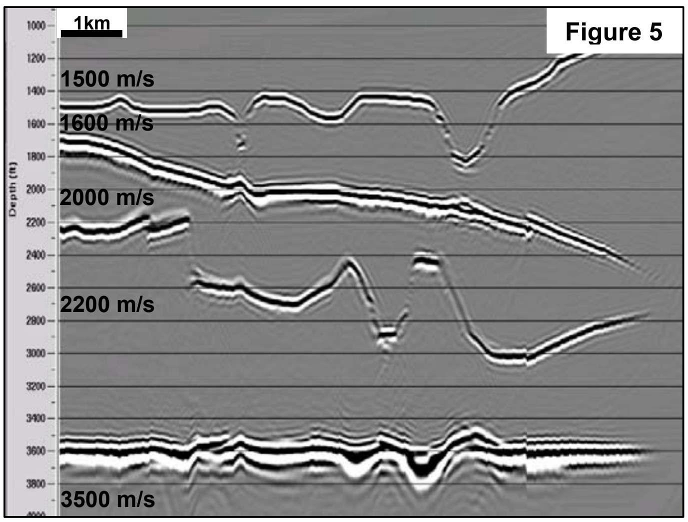
Time migration only seeks to “move” (migrate) recorded data closer to their true spatial positions, and we accept that time migrated positions are only approximate at best (Hubral, 1977; Black & Brzostowski, 1994; Tieman, 1986). Conversely, depth migration does try to create images in their true spatial location when anisotropy and velocity variation are correctly accounted for. Figure 3 shows a 2D synthetic model of a deepwater environment with a corrugated sea bed, with constant velocity layers. The preSDM result is shown in Figure 4 and the preSTM result in Figure 5 (converted to depth for comparison). The failure of time migration to correctly obey Snell’s law results in unacceptable distortion of the deeper flat layer.



Interval velocity model used to create and migrate deep water synthetic data



Kirchhoff preSDM of deep water synthetic data with correct interval velocity Model



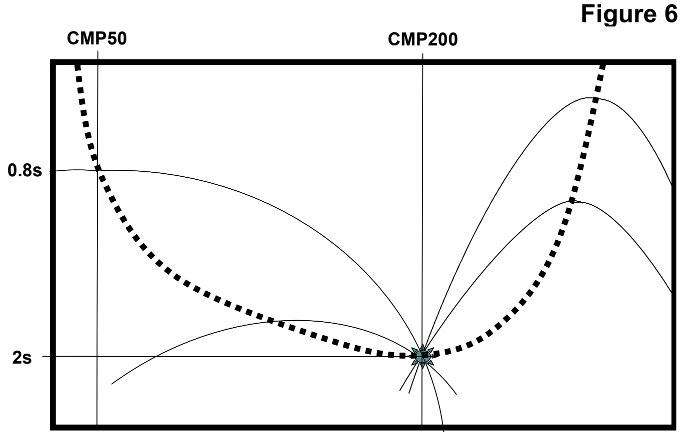
Kirchhoff preSTM of deep water synthetic data (converted to depth) with the RMS version of the correct model. The deepest event should be flat.

The migration response

It is instructive to introduce some more terminology here: namely the migration operator and the migration impulse response. For example, for a 2D time migration, the basic migration operator for each time sample is a symmetric arc which for zero offset source and receiver separation is circular, and for non-zero separation is elliptical. In 3D the response would be a hemispherical bowl and an elongated ellipsoid, respectively. If we ran this time migration, say, on a zero offset plane with laterally varying velocity, at each CMP location the operator would be a circular arc, but the radius of this arc would change for each CMP location, as the arc's radius is proportional to the velocity at that CMP location. The impulse response would be the sum off all such arcs, and what we would see would be the envelope of all such arcs which can have an asymmetric shape. So, it is important to note that although the time migration *operators* are individually symmetric, the overall *impulse response* will not be if the velocity varies laterally.

Alternatively, the process of building the impulse response could be described as a sum along a diffraction trajectory which places the result of this sum at the apex. The curvature of this diffraction trajectory would also change shape if the velocity varies laterally. In Figure 6 we see a single live input sample on a 2D zero-offset plane at CMP location 200 and two-way-time 2s. The output migrated image of this single live sample will be formed by summing along all possible hyperbolic diffraction trajectories that fit in this 2D offset plane: in the simplest time migration case the shape of these trajectories is determined by the rms velocity to the apex of the diffraction. For example, the trajectory with apex at CMP 50, would collect and add all samples along this hyperbolic corridor, and place the result at the apex at 800ms two-way travel time. The majority of such trajectories for this particular input data will only add zeroes together: the only live contributions resulting from this process will be for hyperbolic diffraction trajectories whose diffraction tails happen to intercept the single live sample. When all possible output sums have been computed, the locus of the results constitutes the migration impulse response, and would be symmetric for a constant velocity medium (and also for a 1D laterally invariant velocity function which changes only vertically), but would be asymmetric for laterally varying velocity.

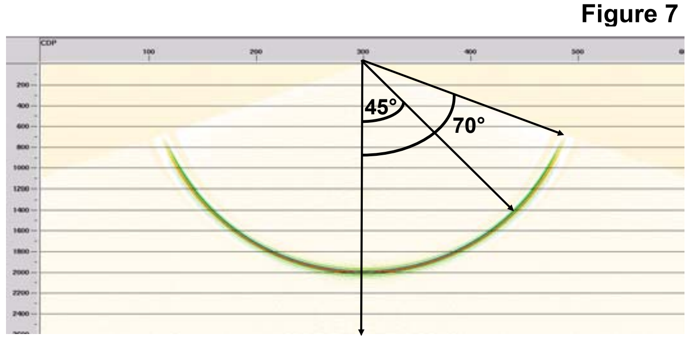
Figure 6



Cartoon showing the principle of summing along a suite of hyperbolic corridors to form the migration output. Any energy captured in a given hyperbolic corridor is placed at the vertex of that corridor to constitute its output contribution. The thick dashed line shows the locus of all such vertices.

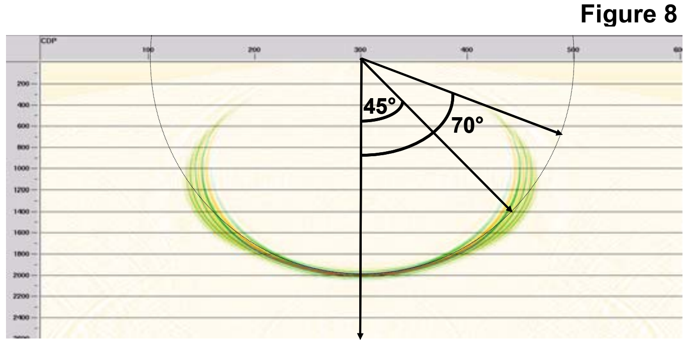
All migration algorithms are implementations of an approximate solution of the wave equation, and one or more of these approximations usually have the effect of limiting the maximum dip that can be accurately reconstructed in the output image. For example, in a Kirchhoff scheme, we select the maximum dip that the ray tracing will be performed for (described further in Part II). If we have signal in the input data emanating from reflectors with steeper dips, they would be effectively filtered out of the resulting image by this dip limitation in the ray tracing. The following Figures show 2D zero offset impulse responses computed for a constant velocity of 2km/s (hence a time and depth image will appear the same, and the response should be semi-circular), with a 50Hz wavelet at 4ms input sampling and a 10m inter-trace distance. On the responses, are denoted the 45º and 70º dip angles. Figure 7 is the Kirchhoff result with an explicit dip limit of 70º whilst for comparison the 15º finite difference (FD) result is shown in Figure 8 (with the correct semi-circular response superimposed in yellow). This latter class of algorithm was common throughout the 1980's but is no longer used due to its assorted artefacts. It is clear for this last algorithm that the equations used to represent the semi-circular wavefronts are not very circular, and do not simply end at some requested dip limit, but continue to create an output beyond the useful parts of the response. Such FD schemes do not explicitly limit the dip of the operators, but are progressively more in error at steeper dips. So in that case, the steeper events may be visible in the output image, but would be systematically mispositioned and dispersed.

Figure 7



dx=10m dz=5m v=2000m/s
Kirchhoff 70 deg
Kirchhoff migration impulse responses with an explicit dip limit of 70º

Figure 8



dx=10m dz=5m v=2000m/s
Implicit FD 15 deg
Finite difference migration impulse response with a 15º implicit FD approximation

Classes of migration: differential versus integral techniques or waves versus rays

There are two categories of theoretical description underpinning migration algorithms (summarized in Table 1), both of which are numerical solutions to the wave equation. These two categories are the integral methods (including Kirchhoff and beam techniques), and the differential methods (such as finite differencing and phase-shift techniques, which use wavefield extrapolation to solve the migration equations (these include reverse time migration (RTM) which despite its name is a type of depth migration, and wavefield extrapolation migration (WEM), also referred to by some as being 'wave equation migration', which is a bit misleading as all the methods attempt to solve the wave equation). Both time and depth migration can be performed with either integral (ray) or differential (wavefield extrapolation) techniques.

This description using integral and differential techniques encapsulates the concepts of rays and waves, respectively. As sound propagates through the earth, it does so along an expanding wavefront, which would look something like a hemispherical bowl that was continuously expanding, with the amplitude at the expanding wavefront in general decreasing as it spread-out. As with a ripple spreading-out on the surface of a pond, there will be a characteristic wavelet spanning the leading edge of the ripple. For a constant sound-speed medium the wavefront will be an expanding hemisphere. When the velocity of sound in the medium is not constant, then the wavefront gets distorted in peculiar ways.

Modelling this process, and backing out its effects during migration, can be done by considering the difference in position and amplitude from one depth slice to the next in the earth. The maths of considering the wavefield in this way falls under the category of wavefield extrapolation or differential techniques. An alternative description of the expanding wavefront would be to consider the normal to the expanding wavefront and to plot (or track) the evolution in time of these normal vectors. These vectors are described as 'rays' and give an indication of the direction of motion of the wavefront, and also the arrival times of the wavefront along the associated ray-path. In their simplest forms, rays do not inherently convey information about phase and amplitude behaviour. Hence as a description, they constitute a gross simplification of the process of wave propagation. Ray description can tell us how long it takes a wavefront to travel from one point to another, and/or the direction the wave moves in. This information is sufficient to perform forward modeling (i.e. to make synthetic data) and also to perform a migration. However, it should be emphasised that contemporary ray-based migration and modelling schemes also include techniques for sensibly estimating amplitude, by considering the behaviour of neighbouring rays as well as the main 'central' ray (Cervény, 1981).

Table 1: Integral versus Differential Methods

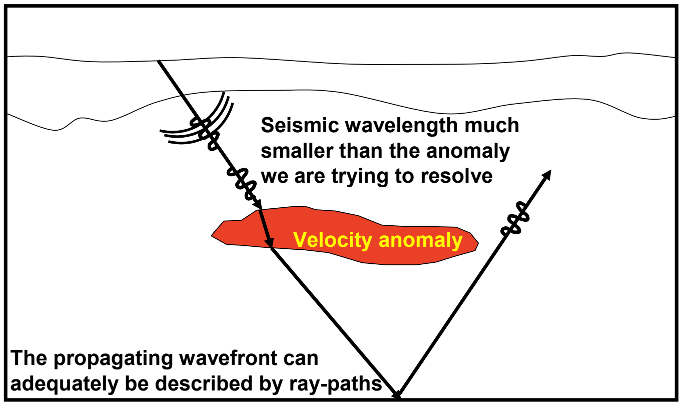
Integral or Ray-based Methods	Differential, Extrapolation or Continuation Methods
- Kirchhoff, Gaussian beam, & fast (controlled) beam are the best known. Usually implemented in the time domain, but can be in the frequency domain. Distinguishing feature is separation of calculation of travel times from imaging: thus a subset of the image can be computed without needing to image the entire volume	- Finite difference wavefield continuation is the best known, in conjunction with 'phase shift plus corrections'. Each depth slice of the wavefield is computed from the previously computed slice, thus the entire image volume needs to be formed. Dip response is dependent on the order of the expansion used (thus potentially costly)
Strengths: - delivers sub-sets of the imaged volume, including offsets or angles (thus cost effective for iterative model building) - good dip response	Strengths: - images all arrivals - simpler amplitude treatment - can be extended to two-way solutions of the wave equation (e.g. RTM)
Weaknesses - Inherently kinematic (only approximate amplitude treatment) - Kirchhoff usually only delivers one arrival path but beam can handle multi-pathing - velocity field coarsely sampled for travel time computation, then arrival times interpolated back to seismic spacing, which can mis-represent rugose high velocity contrast boundaries (such as top salt)	Weaknesses - images whole volume (thus costly) - obtaining good dip response is expensive - does not readily produce pre-stack data - thus difficult to achieve cost-effective iterative model building without 'restrictive' assumptions (eg mono-azimuth)

(adapted from: Jones & Lambaré, 2003)

Velocity scale length

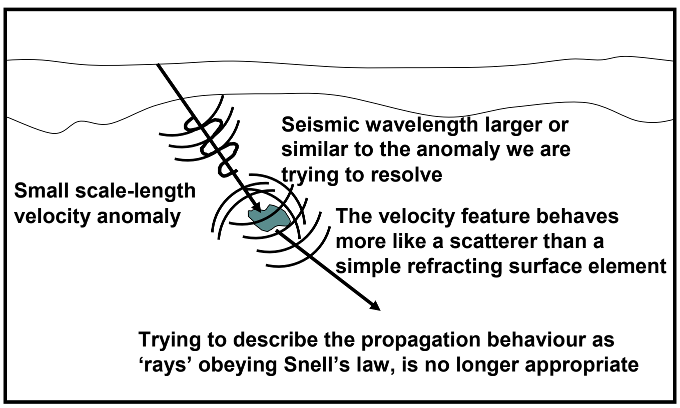
Velocity variation can be classified on the basis of scale length of the variation as compared to the wavelength of the seismic wavelet. If the velocity scale length is much greater than the seismic wavelength, then ray-based techniques for model building (such as conventional tomography) and imaging can resolve the features. If not then this (high frequency) ray approach is inappropriate, as diffraction (scattering) phenomena will predominate, and then waveform tomographic model building and wavefield continuation (differential) imaging techniques ideally need to be used. Figure 9 shows the situation with a velocity anomaly whose physical dimensions are much larger than the seismic wavelength. In this case, describing the propagating wave-front with representative 'rays' (normal to the wave-front) is acceptable as Snell's law adequately describes the refractive and reflective behaviour at the interfaces of the anomalous velocity region. Conversely, once the velocity anomaly is of similar scale length to the seismic wavelet (as shown in Figure 10), then diffraction phenomena dominate, as it is then scattering which governs the behaviour of the wave-front. In this case, rather than just considering a ray description of the events, we need to use the wave equation to estimate how the waveform will propagate (e.g. Pratt, et al., 1996; Sirgue & Pratt, 2002).

Figure 9



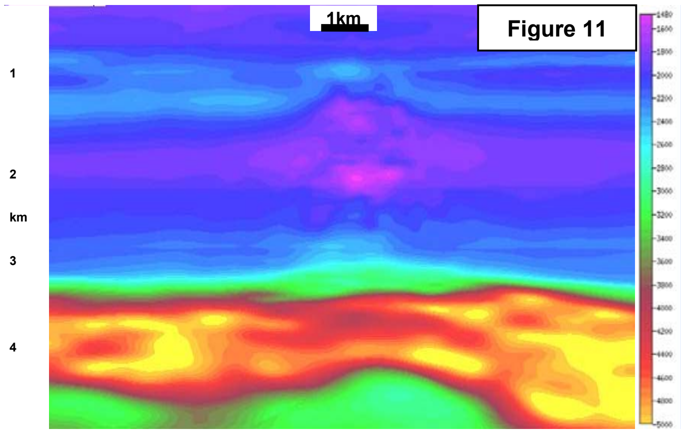
Resolution scale length - velocity anomaly scale length greater than the seismic wave length - ray theory works

Figure 10

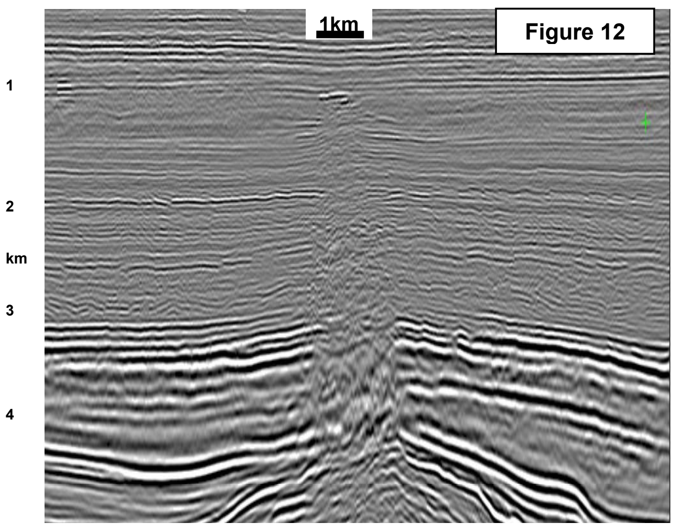


Ray-based methods (Kirchhoff, beam) using the 'high frequency approximation' begin to fail. Resolution scale length - velocity anomaly scale length comparable to the seismic wave length - ray theory fails and diffraction (scattering) theory is better for describing the phenomenon.

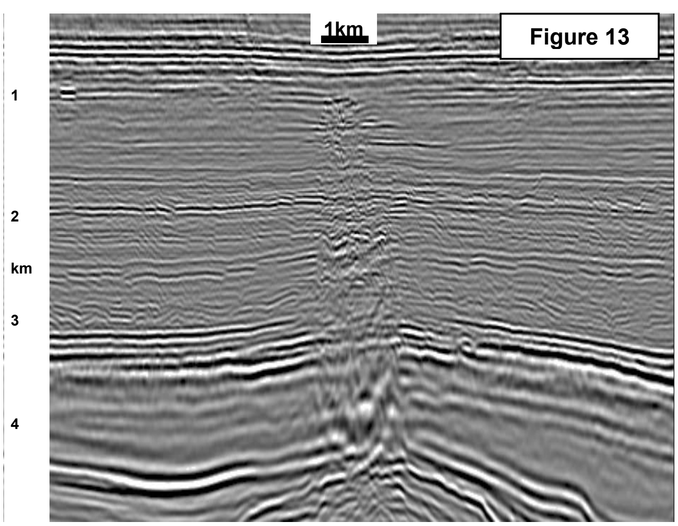
Kirchhoff and beam techniques both have both a good dip response and handle lateral velocity variation very well, as long as the spatial wavelength of these changes is much longer than the seismic wavelength. However, for lateral velocity variation on a length scale similar to the seismic wavelengths, ray techniques are no longer appropriate. In Figures 11, 12 and 13, we see an interval velocity model and some simple looking relatively flat data, but with a series of small gas charged lenses in the overburden (courtesy of ConocoPhillips Norway). The velocity model was constructed by constraining high-resolution ray-based tomography with very dense well control (more than 100 wells were available over the crestal structure). The result in Figure 11 shows a 3D anisotropic Kirchhoff preSDM, while Figure 12 shows the result of a wavefield continuation migration. The latter technique has honoured the small scale high velocity contrast features in the model. The gas lenses which are about 200m wide, have a velocity of about 1400m/s in a background velocity of about 2000m/s, and hence a ray tracing procedure has difficulty preserving this detail.



Flat data with gas lenses: velocity model derived using ray-based tomography with substantial well constraint. Data courtesy of ConocoPhillips Norway



Flat data with gas lenses: the Kirchhoff ray tracing cannot honour the short wavelength velocity anomaly.



Wavefield extrapolation migration with the same input data and model is better able to image the small features

Domains of application
Time and depth migration techniques can be applied in various 'domains': The domain of application is a separate issue from the type of description we are using (i.e. waves or rays). The common domains are time-space (t,x,y), frequency-space (f,x,y), frequency-wavenumber (f,k_x,k_y), and zero-offset-time and ray-parameter (tau-$p_x p_y$). The reason for selecting one domain over another is primarily to exploit some property of that domain that will save computation time or reduce a class of noise. For example, if we had data with usable signal bandwidth of 5-55Hz, then with an (f,x,y) implementation of wavefield extrapolation we can reduce cost by migrating only up to 55Hz, and ignoring all frequencies above this. If the same class of algorithm was implemented in the (t,x,y) domain, we could not exploit this cost saving, nor could we readily exclude any high frequency noise in the input data during migration.

In addition to the domain of application for the migration, we also have the issue of input data ensemble to consider. The algorithm in use (e.g. a wavefield extrapolation in the (f,x,y) domain) can be applied to the input data in different sort orders e.g common shot, common receiver, common offset, etc, and there are various reasons why we might want to use one sort order over another (e.g. ease of throughput for data access, or adherence to the requirements of some algorithmic approximation).

Evolution of wavefield extrapolation migration schemes
Prior to the early 1990's the limitations on affordable availability of computer power effectively limited migration to the post-stack domain. At that time, a common means of performing both post-stack time and depth migration (postSTM and postSDM) of 3D seismic data was via the use of frequency domain implicit finite difference (FD) algorithms, first introduced in a geophysical context by Claerbout (1976). To facilitate solution of the 3D wave equation with FD schemes, a technique called 'splitting' was invoked, whereby an independent 2D solution was implemented for the in-line (x) and cross-line (y) directions. This involved separating a square-root equation (containing the spatial variables x and y) into two independent square root terms one for each of the two spatial variables. This splitting, or separation of the x and y components in the data, resulted in 'numerical anisotropy' - yielding an impulse response which did not possess the requisite circular x-y section for a constant velocity medium. (The name arises by analogy with physical anisotropy, which results in waves propagating at different velocities in different directions, resulting in a non-spherical wave front).

Each resulting square root term was then approximated by a series expansion, the truncation of which led to an incorrect positioning of energy beyond a certain dip in the migrated output. A better dip response can be obtained by using higher-order expansions in approximating the square root term, but this greatly increases the cost of the migration. Such series expansion approximations do not have an inherent dip limiting cut-off for the steeper dips where the approximation is no longer valid, but simply misposition energy beyond this limit (Figure 8). Consequently, a form of noise was introduced appearing as energy travelling at impossibly high velocities for a given propagation angle (terms which would have given rise to a negative term within the square root).

Also, using finite differencing techniques to solve the second-order differential term of the wave equation results in a slight mispositioning of energy as a function of frequency, with respect to the sampling grid of the data. This gives rise to a phenomenon resembling dispersion, in that different frequencies appear to travel at different speeds. During migration a single dipping event will split into a suite of different events each of different frequency content and dip (Diet & Lailly, 1984). However, the introduction of explicit continuation schemes, free from the FD artefacts, led to steep dip high fidelity postSDM algorithms seen routinely in use by the mid 1990's (Hale, 1991b, Soubaras, 1992).

In addition, migration prior to the late 1990's was isotropic. However, if attempting to deal with non-elliptic anisotropic media, we face an additional problem with FD solutions of the acoustic wave equation (i.e. after dropping the shear terms in the elastic wave equation) as anisotropic behaviour is essentially an elastic phenomenon, so is not correctly dealt with in an acoustic migration scheme. Hence another class of algorithm noise appears for non-elliptic anisotropy for the FD acoustic approximation (Bale, 2007).

For the most part, these FD and explicit techniques fell into abeyance in the mid to late 1990's as post-stack migration was superseded by Kirchhoff pre-stack migration, and depth migration started to emerge as a serious imaging technique. 3D pre-stack imaging became computationally feasible due the appearance of efficient first-arrival travel-time solvers for depth imaging and more approximate travel time estimators for time imaging. Furthermore, the ability of Kirchhoff (and other integral methods) to produce limited subsets of the image and pre-stack migrated gathers made industrial application affordable, especially given the fact that we had to apply depth imaging methods iteratively to build velocity models. Thereafter, industrial efforts went towards the improvement of Kirchhoff migration, both in terms of amplitudes and handling different branches of the arrival times.

In the early 2000's, as computer costs became less of an issue, we saw a resurgence in one-way WE techniques, but for the pre-stack domain (I'll describe the notion of 'one-way' a bit later). Wavefield extrapolation implementations of the one-way scalar wave equation are relatively simple to write compared to a Kirchhoff scheme, but in principle are more costly; the extra cost may be prohibitive if many iterations are needed for construction of the velocity model. With Kirchhoff migration, it is routine to output the data for model update sorted by surface offset (resulting in familiar-looking migrated gathers, variously referred to as common reflection point (CRP) or common image point (CIG) gathers). For shot domain and other wavefield extrapolation techniques, we require various additional approximations to produce gathers for velocity analysis, as these schemes do not inherently produce a pre-stack output domain, but rather the zero-offset image only (e.g. Faye and Jeannot, 1986; Sava and Vasconcelos, 2008). This evolution of industrially available migration algorithms is summarized in Table 2.

Table 2: time-line for evolution of industrial techniques

Period in use as primary deliverable	Technique	Common domain & type of application
1975- 1988	2D postSTM	Finite Difference (FD) (x,t) & (x,f) Initially with 30º, then 45º and later 60 º dip limits
1980-1988	2D postSDM	FD (x,f) Initially 45º and later 60º dip limits
1985-1995	3D postSTM	FD (x,y,f) Initially with 45º and later 60º dip limits
1990-2001	DMO + 3D zero-offset constant velocity preSTM, followed by a de-migration of the stack and then 3D postSTM	Constant velocity phase shift (Stolt) zero offset preSTM, and subsequent de-migration, in conjunction with FD (x,y,f) postSTM
1990-1995	2D full-offset preSDM	FD focussing analysis interactive (x,f)
1993-1997	DMO + 3D zero-offset constant velocity preSTM, followed by a de-migration of the stack and then 3D postSDM	Constant velocity phase shift (Stolt) zero offset preSTM, and subsequent post-stack de-migration, in conjunction with FD (x,y,f) postSDM
1995 - present	Full-offset v(x,y,z) 3D preSDM	Kirchhoff (x,y,z) isotropic
2000-2003	Full-offset v(x,y,t) 3D preSTM	Kirchhoff (x,y,t) straight ray
2002-present	Full-offset v(x,y,t) 3D preSTM	Kirchhoff (x,y,t) curved and turning ray & anisotropic
2000-present	Full-offset v(x,y,z) 3D preSDM	Isotropic wavefield extrapolation (WE), either with for example: FD, SSFPI, & non-WE beam
2000 - present	Full-offset v(x,y,z) 3D preSDM outputting gathers	TTI Kirchhoff (x,y,z) anisotropic turning ray
2005- 2008	Full-offset v(x,y,z) 3D preSDM outputting gathers	VTI wavefield extrapolation, either with for example: FD, SSFPI, and alternatively non-WE beam
2006- present	Full-offset v(x,y,z) 3D preSDM	VTI two-way wavefield extrapolation using reverse time migration, or two-pass one-way extrapolation
2008- present	Full-offset v(x,y,z) 3D preSDM outputting gathers	VTI beam or two-way wavefield extrapolation using reverse time migration
2009- present	Full-offset v(x,y,z) 3D preSDM outputting gathers	TTI beam or two-way wavefield extrapolation using reverse time migration

(adapted from Jones et al., 2008)

It should still be kept in-mind however, that all the schemes in use within the industry today are solutions of the acoustic wave equation hence none of them deal with mode conversion, transmission and reflection energy partitioning, and they also ignore absorption (Q).

Part II

In the concluding part of this tutorial, the aspects relating to ray based migration, and associated noise will be covered, as well as multipathing and two way propagation.

References

Abma, R., Sun, J. and Bernitsas, N., 1999, Antialiasing methods in Kirchhoff migration: Geophysics, Soc. of Expl. Geophys., 64, 1783-1792.

Babich, V.M., and Popov, M.M., 1989, Gaussian beam summation method (survey). Radiophysics and Quantum Electronics, 32, 1063-1081.

Bale, R., 2007, tti vs vti 69th Mtg. Eur. Assn. Geosci. Eng.

Bancroft, J. C., and H. D. Geiger, 1994, Equivalent offset CRP gathers: 64th Annual International Meeting, SEG, Expanded Abstracts, 672–675.

Cervény, V., 1981, Ray tracing in a vicinity of the central ray, SEP-28, 39-48

Červený, V., M. M. Popov, I. Pšenčík, 1982, Computation of wave fields in inhomogeneous media — Gaussian beam approach, Geophysical Journal International 70 No.1, 109–128

Claerbout, J.F., 1992, Earth Soundings Analysis: PVI, Blackwell Scientific Publications.

Faye, J. P. and Jeannot, J. P., 1986, Prestack migration velocities from depth focussing analysis: 56th Ann. Internat. Mtg.: Soc. of Expl. Geophys., Session:S7.6

Gray, S. H., 1992: Frequency-selective design of the Kirchhoff migration operator: Geophys. Prosp., Eur. Assn. Geosci. Eng., 40, 565-572.

Gray, S. H., 2009, Beam migration, The Leading Edge,

Hubral P., 1977, Time migration; some ray theoretical aspects, Geophysical Prospecting, 25, no. 4 , 738 - 745

Jones, I. F., 2010, An Introduction to Velocity Model Building, EAGE publications, 296p.

Jones, I. F., and G. Lambaré, 2003, Wave equation versus ray based imaging: First Break, 21, no. 2, 11–13.
Jones, I.F., Bloor, R.I., Biondi, B., Etgen J.T., 2008, Pre-stack depth migration and velocity model building, SEG Geophysics Reprints, 25, Series Editor: Michael A. Pelissier.
Lazaratos, S. K., and J. M. Harris, 1990, Radon transform/Gaussian beam migration: 60th Annual International Meeting, SEG, Expanded Abstracts, 1314–1317.
Lumley, D. E., Claerbout, J. E. and Bevc, D., 1994, Anti-aliased Kirchhoff migration, 56th Mtg.: Eur. Assn. of Expl. Geophys., Session:H027
Pelissier, M.A., Hoeber, H., van de Coevering, N., Jones, I.F., 2007, Classics of Elastic Wave Theory, SEG Geophysics Reprints, 24, Series Editor: Daniel A. Ebrom, 534 pages.
Popov, M. M, 1982a, A new method of computation of wave fields using Gaussian beams: WaveMotion, 4, 85-97.
Robein, E., 2003, Velocities, time-imaging and depth-imaging: principles and methods: EAGE Publications.