Efficient Q-RTM in anisotropic TTI media with phase and amplitude compensation

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Summary

Starting from the complex phase velocity, in this paper, we derive a system of 2nd order wave equations in 3D visco-acoustic TTI media that can be employed in Reverse Time Migration (RTM) to simultaneously compensate for phase and amplitude distortions. Unlike most available wave equations with Q compensation in the literature, which have either fractional order Laplacian operator or higher order spatial derivatives, the wave equations derived here can be regarded as an extension of the system of 2nd order wave equations in acoustic TTI media to that in visco-acoustic TTI media. These equations can be implemented efficiently using finite-differences in the time-space domain. We demonstrate the effectiveness of the proposed solution for modeling and migration with synthetic and field data.

Introduction

As seismic activities move more toward production than exploration, the preservation of Amplitude Versus Offset (AVO) behavior is of great importance. Seismic attenuation, which is often quantified by the Q factor, is one of the main factors that affects the resolution of images and amplitude variations of reflections with angles or offsets. Compensating for Q effects has been an active topic for some time in the industry (*e.g.,* Kjartansson, 1979). Compensating for phase and amplitude distortions can be applied to the unmigrated data in the time domain assuming a relatively simple Q model. Such compensation is inaccurate in the presence of complex geology where a model driven Q-compensation becomes a requirement to reduce the uncertainty of the final migrated images.

More recently, solutions for Q compensation have been proposed during migration. In an isotropic medium, Zhang *et al.* (2010) proposed an amplitude-only Q-RTM that can be applied in the time-space domain; a similar idea was later extended to TTI media by Suh *et al.* (2012). Hu *et al.* (2016) and Zhou *et al.* (2020) developed imaging algorithms in isotropic and TTI media based on the simultaneous phase and amplitude Q modeling method proposed by Blanch *et al*. (1995).

In this paper, we derive a new system of 2nd order wave equations in 3D TTI visco-acoustic media to simulate Q effects for both phase and amplitude distortions, which can be readily implemented in the time-space domain using finite-differences; a similar system of wave equations can be constructed for Q-RTM, where, to compensate for the phase and amplitude distortions, the equations can be constructed by changing the signs (from positive to negative) of certain parameters and variables in the modeling equations. We demonstrate the proposed approach for Q-modeling and Q-RTM with synthetic and field data examples.

Method

In wavefield propagation with Q attenuation, the complex P-wave phase velocity can be presented as follows (Blanch *et al*., 1995)

$$v_p^A(\omega) = v_p(\omega)\left(1 + \frac{i \cdot sgn(\omega)}{2Q(\omega)}\right) \approx v_p(\omega)\sqrt{1 + iQ^{-1}(\omega)} \quad (1)$$

where the superscript *"A"* stands for an attenuation, ω is the angular frequency, $v_p^A(\omega)$ is the frequency dependent complex phase velocity, and $v_p(\omega)$, the real part of $v_p^A(\omega)$, is the P-wave phase velocity in a visco-acoustic medium.

Blanch *et al*. (1995) presented the formulation of frequency dependent inverse Q as follows,

$$Q^{-1}(\omega) = \sum_{l=1}^{L} \frac{\omega\tau_{\sigma_l}\tau}{1+\omega^2\tau_{\sigma_l}^2} \quad (2)$$

where "L" is the Standard Linear Solid (SLS) number (Day and Minster, 1984) and

$$\tau = \frac{\tau_{\varepsilon_l}}{\tau_{\sigma_l}} - 1 = \frac{\tau_{\varepsilon_l} - \tau_{\sigma_l}}{\tau_{\sigma_l}} \quad (3)$$

where τ is dimensionless, and τ_{ε_l} and τ_{σ_l} are the strain and stress relaxation times for the l-th SLS, respectively.

Substitute Eq. (2) into (1), we obtain the following Equation:

$$v_p^A(\omega) \approx v_p(\omega)\sqrt{1 + \sum_{l=1}^{L} \frac{i\omega\tau_{\sigma_l}\tau}{1+\omega^2\tau_{\sigma_l}^2}}. \quad (4)$$

The frequency dependent phase velocity $v_p(\omega)$ can be represented in the following form (Blanch *et al*., 1995),

$$v_p(\omega) = v_p^0\sqrt{1 + \sum_{l=1}^{L} \frac{\omega^2\tau\tau_{\sigma_l}^2}{1+\omega^2\tau_{\sigma_l}^2}} \quad (5)$$

where v_p^0 is the P-wave phase velocity in an acoustic medium ($Q = +\infty$).

Thus, Eq. (1) can be approximately expressed as follows:

$$v_p^A(\omega) \approx v_p^0 \sqrt{1 + \sum_{l=1}^{L} \frac{\omega^2 \tau \tau_{\sigma_l}^2}{1+\omega^2 \tau_{\sigma_l}^2} + \sum_{l=1}^{L} \frac{i\omega \tau_{\sigma_l} \tau}{1+\omega^2 \tau_{\sigma_l}^2}} . \quad (6)$$

Assuming $\omega^2 \tau_{\sigma_l}^2$ being a small quantity, Eq. (6) is further simplified as follows:

$$v_p^A(\omega) \approx v_p^0 \sqrt{1 + \sum_{l=1}^{L} \omega^2 \tau \tau_{\sigma_l}^2 + \sum_{l=1}^{L} i\omega \tau_{\sigma_l} \tau} . \quad (7)$$

The above complex phase velocity $v_p^A(\omega)$ then can be substituted into one of the commonly applied TTI wave equations in acoustic media like that proposed by Fletcher *et al.* (2009). After some mathematical derivations, the system of wave equations in visco-acoustic TTI media can be obtained as follows,

$$\frac{\partial^2 p}{\partial t^2} = (1 + L\tau)\left[v_{px}^2 H_2(p) + \alpha v_{pz}^2 H_1(q)\right] + \sum_{l=1}^{L} u_l + v_{sz}^2 H_1(p - \alpha q) \quad (8a)$$

$$\frac{\partial^2 q}{\partial t^2} = (1 + L\tau)\left[\frac{1}{\alpha} v_{pn}^2 H_2(p) + v_{pz}^2 H_1(q)\right] + \sum_{l=1}^{L} v_l - v_{sz}^2 H_2\left(\frac{1}{\alpha} p - q\right) \quad (8b)$$

and

$$\frac{\partial u_l}{\partial t} = -\frac{\tau}{\tau_{\sigma_l}}\left(v_{px}^2 H_2 p + \alpha v_{pz}^2 H_1 q\right) - \frac{1}{\tau_{\sigma_l}} u_l \quad (9a)$$

$$\frac{\partial v_l}{\partial t} = -\frac{\tau}{\tau_{\sigma_l}}\left(\frac{1}{\alpha} v_{pn}^2 H_2 p + v_{pz}^2 H_1 q\right) - \frac{1}{\tau_{\sigma_l}} v_l \quad (9b)$$

where p and q are the pressure and auxiliary wavefields in TTI media, respectively, u_l and v_l are memory variables, H_1 and H_2 are the differential operators acting in the vertical and horizontal directions, respectively, v_{px} is the P-wave phase velocity in the symmetry plane, v_{pz} is the P-wave phase velocity in the direction normal to the symmetry plane, v_{pn} is the P-wave normal moveout velocity, and v_{sz} is the SV-wave phase velocity in the direction normal to the symmetry plane.

Compared to most existing wave equations with *Q* compensation, where the *Q* effects are presented with either fractional order Laplacian operator or higher order (*e.g.*, the 4th order) spatial derivatives (Kjartansson, 1979; Hu *et al.*, 2016; and Zhou *et al.*, 2020; *etc.*), Eqs. (8) and (9) are a system of 2nd order wave equations, which is much like the one in acoustic media that has been widely applied in the seismic community (Fletcher *et al.*, 2009). Thus, its implementation is straightforward and efficient using finite-differences.

To derive a system of wave equations for reverse time migration in visco-acoustic media, where the *Q* effects should be compensated, one needs to modify Eqs. (8) and (9) by changing the signs of parameter τ and variables u_l, v_l and t, from positive to negative. In addition, it is worth mentioning that a stabilization step should be needed in practice to avoid overcompensating the *Q* effects during the wave back-propagation.

The system of wave equations in (8) and (9) simulates both the phase and amplitude distortions simultaneously. These two equations can be reduced to the case that simulates only amplitude attenuation as a special case. When $\omega^2 \tau_{\sigma_l}^2$ is a small quantity, Eq. (2) can be approximately simplified to

$$\frac{1}{Q_r \omega_r} \approx \sum_{l=1}^{L} \left(\tau_{\sigma_l} \tau\right) \quad (10)$$

where ω_r is the reference angular frequency and Q_r is a reference *Q*-value. With the above Eq. (10), Eq. (8a) can be reduced to the following Equation:

$$\frac{\partial^2 p}{\partial t^2} + \frac{\omega_r}{Q_r} \frac{\partial p}{\partial t} + {\omega_r}^2 p = 0 . \quad (11)$$

Eq. (11) is the approximated version of the amplitude-only attenuation equation presented in Kjartansson (1979), Zhang *et al.* (2010) and Suh *et al.* (2012).

Numerical Examples

2D phase and amplitude simultaneously affected Q modeling

We conducted a 2D *Q* modeling test in an isotropic medium using Equation 8(a) for validation. Figure 1(a) shows a 2D modeling result without *Q*, while (b) is the 2D *Q* modeling result with *Q*=10000, (c) is the 2D *Q* modeling result with a much smaller *Q* of 20, and (d) is the result of the 2D amplitude-only *Q* modeling with *Q* being 20.

As expected, in the case of *Q*=10000, its impact is negligible, the results of (a) and (b) are essentially identical to each other in terms of wavelet, phase, amplitude, and spectrum. In (c), for *Q*=20, large phase distortions are observed comparing the two side-lobes of the wavelet in (c) to (a). Meanwhile, the amplitude decay, and suppression of high-frequency contents are also obvious in (c). In contrast, for the case of amplitude-only dissipation in (d), amplitude decay is observed as well as suppression of high frequency contents with an unaffected phase.

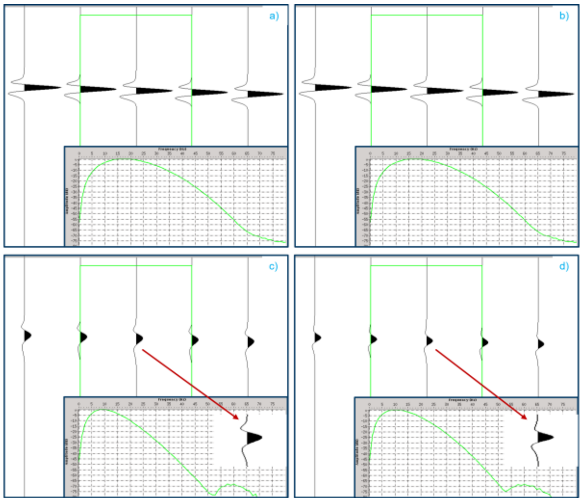

Figure 1: (a) 2D standard modeling without *Q*, (b) 2D phase and amplitude simultaneously affected *Q* modeling with *Q*=10000, (c) 2D phase and amplitude simultaneously affected *Q* modeling with *Q*=20, and (d) 2D amplitude-only dissipated *Q* modeling with *Q*=20.

3D TTI Q-RTM of synthetic data

We conducted a 3D TTI *Q*-RTM test on a 3D synthetic dataset. Figure 2(a) displays the inverse *Q* model used to generate the synthetic data and also used for migration, Figure 2(b) is the standard 3D TTI RTM image using data modelled without *Q* attenuation, Figure 2(c) shows the standard 3D TTI RTM image using data modelled with *Q* attenuation and Figure 2(d) is the 3D TTI *Q*-RTM image using data modelled with *Q* attenuation. Comparing the result in (c) to that in (b), we observe that the amplitudes beneath the shallow *Q* anomaly are damped due to the *Q*-attenuated data. When using 3D TTI *Q*-RTM, the fuzzy amplitude zones are recovered properly in (d).

3D TTI Q-RTM with field data

We applied TTI *Q*-RTM to an OBN field dataset from North Sea and compared the result to that migrated with standard TTI RTM without *Q*. Figure 3(a) shows the inverse *Q* model with *Q* values varying from 25 to 10000 that contains a strong *Q* anomaly beneath the shallow water bottom. Figure 3(b) illustrates the imaging result from the standard TTI RTM without compensating the *Q* effects, while Figure 3(c) shows the result from TTI *Q*-RTM. By comparing these two results, we can see that the phase of the image in Figure 3(c) has been corrected in terms of the depth-positioning and coherence, meanwhile, the resolution and amplitude of the image are significantly improved as well, particularly beneath the *Q*-anomaly as highlighted by the two windows in the Figure. Figure 3(d) is the amplitude-spectral comparisons between the standard RTM image and the *Q*-RTM image in the two windowed areas indicated in Figures 3(b) and 3(c). The red solid curves in both panels represent the amplitude-spectra from *Q*-RTM, while the green solid curves stand for the amplitude-spectra from the standard RTM without *Q*. It is evident from the comparisons that the high-frequency contents have been recovered in the *Q*-RTM image, particularly for the deeper highlighted window.

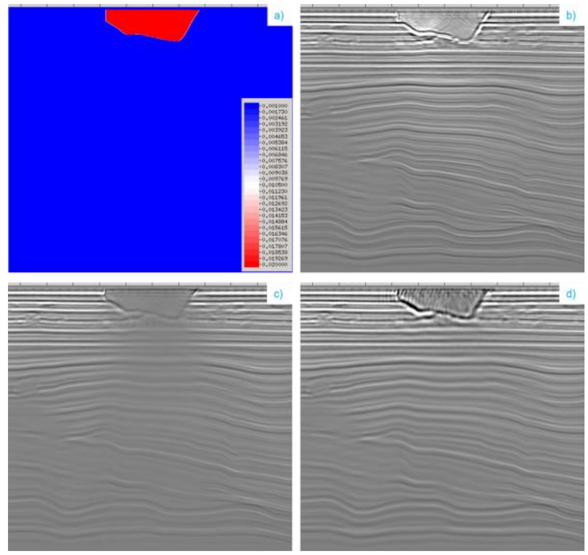

Figure 2: 3D realistic synthetic data: (a) inverse *Q* model, (b) standard 3D TTI RTM image using data without *Q* attenuation, (c) standard 3D TTI RTM image using data with *Q* attenuation, and (d) 3D TTI *Q*-RTM image using data with *Q* attenuation.

Conclusions

Unlike higher order (*e.g.*, the 4th order) spatial derivatives involved *Q* compensation wave equation implementations in the publication, we derive a system of wave equations with the 2nd order spatial derivatives in a visco-acoustic TTI medium that effectively simulates the phase and amplitude distortions, from which we proposed efficient *Q* compensated wave equations for 3D TTI *Q*-RTM to compensate for the distortions of both phases and amplitudes in the data.

The synthetic and field data examples show the improvements to the image from *Q*-RTM compensating for both amplitude and phase effects.

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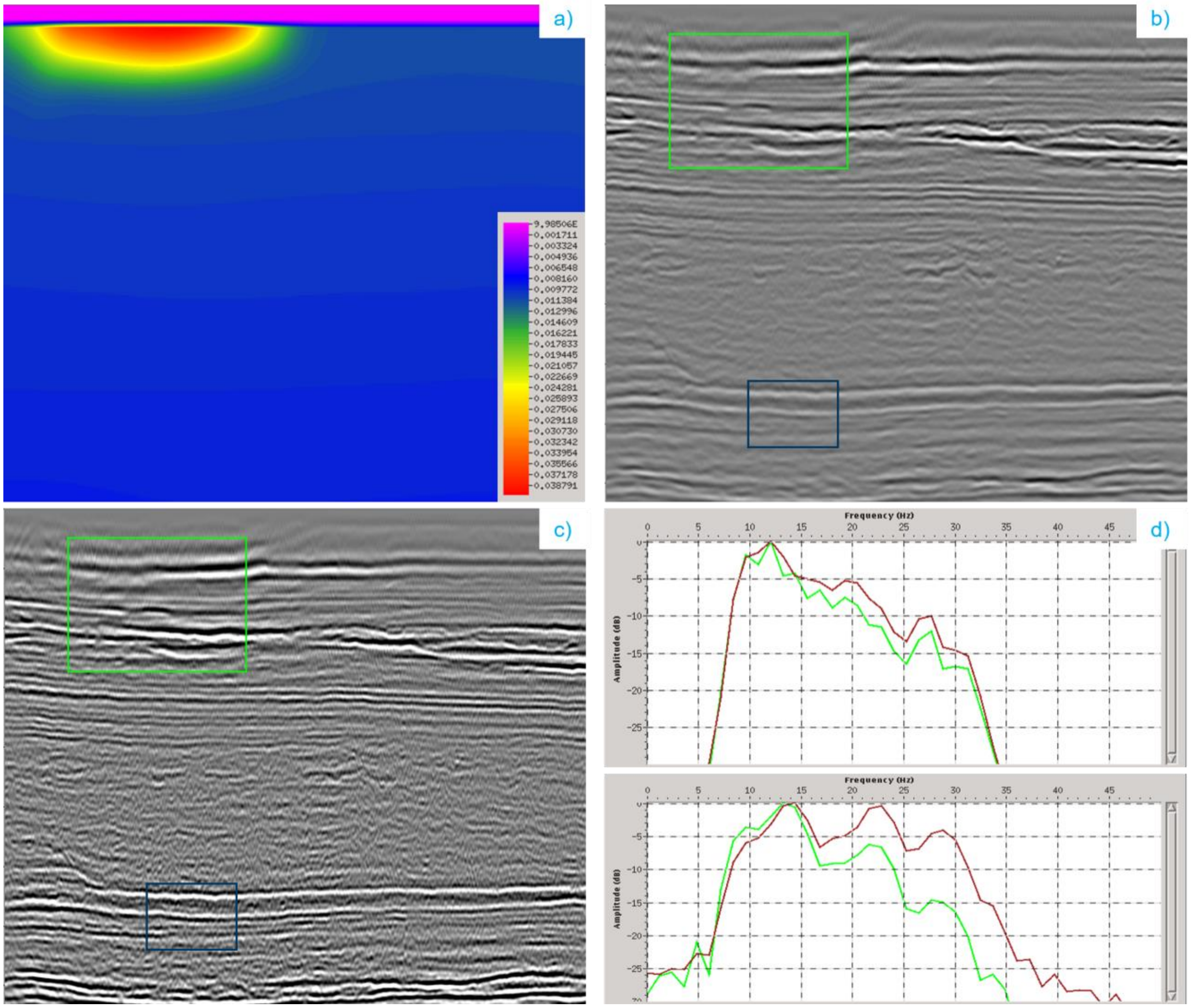

Figure 3: 3D OBN field data from North Sea: (a) inverse *Q* model, (b) standard TTI RTM image without *Q*, (c) TTI *Q*-RTM image, and (d) amplitude-spectral comparisons of the images from the shallow and deep windowed areas. The red solid curves in both panels in (d) stand for the TTI *Q*-RTM, while the green solid curves represent standard TTI RTM without *Q*. Note: The spectra plotted in (d) are separately normalized by their own maxima in each window.