Optimized downward continuation for TTI WEM

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SUMMARY

This paper proposes an improvement in the implementation of the Fourier Finite Difference (FFD) method for the one-way wave equation. Errors associated with FFD can be magnified in media with strong lateral variations in velocity. To reduce the magnitude of these errors, the reference velocity at each depth can be replaced by a velocity function which is a smooth version of the exact model. A phase-shift plus interpolation (PSPI) step is employed to compute the phase shift to this intermediate velocity. To illustrate the accuracy of the method, several migration examples are shown.

INTRODUCTION

High frequency, high resolution seismic images are often required in many geophysical applications. For example, site survey for a shallow section of the earth model needs an image of more than 100 hz, where, in many cases, the structures are relatively simple with mostly flatish stratigraphy. In such cases, the one way wave equation based imaging algorithms are preferred because it is relatively more efficient compared to the two way wave equation based reverse time migration (RTM). In addition, surface multiples in marine environment have been identified complementary to primary reflection for imaging especially for shallow targets. However, suppressing crosstalk between unassociated order of reflections has been one of the challenges in the joint migration of primaries and surface multiples. Imaging using one-way wave equation based methods can generate less crosstalk.

There are many approaches for implementing one way wave equation based migration by downward continuation, from the phase-shift plus interpolation method (Gazdag, 1984) to implicit finite differences based methods (Ristow and Ruhl, 1997). These methods involve two steps: first performs the downward continuation to some constant reference velocity, and the second step adds corrections which account for lateral variation in media properties. The first step is performed in Fourier domain, and the result is exact; the second step is implemented in the spatial domain, and is where errors are introduced in the result. Generally speaking, we might expect that the magnitude of the errors is dependent of the difference between the reference velocity and the actual velocity of the media.

The PSPI method is expected to work well when velocity of the media is relatively smooth laterally; it also enjoys the advantage that one can use multiple reference velocities, and therefore the errors associated with the method can be made smaller (at some additional compute costs). However, it is challenging to implement PSPI in anisotropic media. The finite difference approach, in conjunction with four way splitting and optimization of the coefficients for the finite difference equations, is quite accurate even when velocities change fast, and it can easily account for anisotropy. For these reasons, FFD methods are preffered in current WEM algorithms. However, there are still some residual errors in standard FFD implementations, and these errors tend to be magnified when there is large lateral contrast in the velocity of the media.

In order to reduce the errors associated with the FFD method, we propose a mixed approach, consisting of a PSPI transformation to an intermediate, smooth reference velocity, followed by a FD correction from this reference velocity to the exact velocity. The intermediate model is isotropic, and by choosing it smooth enough, the PSPI step can be made accurate. The anisotropy in the media is fully dealt with by FD. Compared with other approaches of improving the accuracy of FFD, which may require additional FD transformations (Biondi, 2002), or higher order operators (Shan, 2007, Valenciano et al., 2009), the method proposed here has the advantage of being easy to implement and cost-effective, since the PSPI step is quite cheap in comparation with the FD solver.

THEORY

The WEM method is based on the downward continuation equation

$$u(\omega, z+dz) = u(\omega, z)e^{ik_z dz} \ . \qquad (1)$$

In an homogenous media, this equation is implemented exactly in the wavenumber domain, with k_z given by the applicable dispersion relation as a function of ω and k_x, k_y; for example, in isotropic media

$$k_z = \frac{\omega}{v_p}\sqrt{1 - \frac{k_x^2 + k_y^2}{(\omega/v_p)^2}} \ . \qquad (2)$$

The challenge appears when lateral variations in media properties are present; in this case, the vertical wavenumber is usually split into 3 parts:

$$k_z dz \ \simeq \ k_z^0 + \omega\left(\frac{1}{v_z(x)} - \frac{1}{v_0}\right) + \Delta k_z \ . \qquad (3)$$

The first term in the above equation (the phase shift) is again implemented in frequency-wavenumber domain, using a constant reference velocity v_0 (which is typically chosen to be the minimum velocity value at depth z). The second term (split-step) is applied in the frequency-space domain, and its purpose is to correct the phase for waves propagating vertically. (Here v_z is the vertical phase velocity, which is different than v_p for a TTI medium). The last term is an angle dependent correction for downward-propagating waves. In the Optimized Fourier Finite Differences method, this term is approximated using a Padé polynomial of low order, and subsequently converted into a cascading set of implicit finite difference equations which are solved by means of tridiagonal solvers.

From the above equation, it follows that the phase correction due to the FD term is

$$\Delta\phi \;=\; \Delta k_z dz \;=\; (k_z - k_z^0 - \omega(s_z - s_0))dz \;=\; (s\omega dz)P \;, \quad (4)$$

where s is the (vertical) slowness $s = 1/v$. There are several steps involved in evaluating the FD correction:

1. approximate the operator P by a sum of (second order) Padé polynomials:

$$P \;\simeq\; \sum_j \frac{a_j \tilde{k}_j^2 + c_j \tilde{k}_j}{1 - b_j \tilde{k}_j^2 - d_j \tilde{k}_j} \;. \quad (5)$$

In order to insure good accuracy along all azimuths, the sum is usually performed over 4 directions : x, y and the two diagonals; $\tilde{k}_j$ are the scaled wave numbers along the corresponding j axis: $\tilde{k}_j = v_p k_j/\omega$.

2. Approximate the (complex) exponential of the phase shift by a fraction:

$$e^{i\Delta\phi} \;\simeq\; \frac{1 + i\Delta\phi/2}{1 - i\Delta\phi/2} \;. \quad (6)$$

3. Inverse Fourier transform the the k_j terms, which become differential operators $\partial/\partial x_j$. These in turns are evaluated through finite difference operators

$$\tilde{k}_j \;\rightarrow\; -i\frac{v_p}{\omega}\frac{\partial}{\partial x_j} \;\rightarrow\; -\frac{i}{s\omega dx_j}\Delta x_j \;, \quad (7)$$

with Δx_j the FD operator.

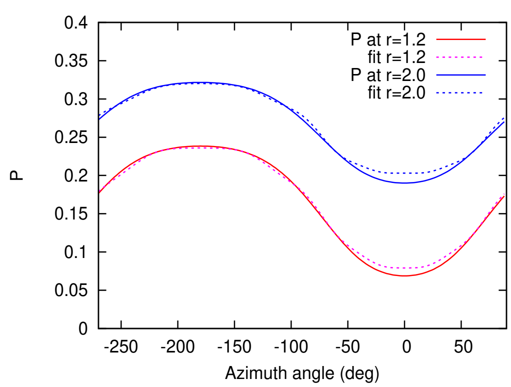


Figure 1: Scaled phase term P as a function of azimuthal angle, at different values for r=v/v_0. The solid lines are the exact values obtained from the dispersion relation, while the dashed lines are the least square fit using Padé polynomials with 4 way splitting.

Most discussion in the literature concentrates on the first term: the Padé approximation for the phase shift in Eq. (5). The coefficients $a, b, \ldots$ are usually estimated by a least squares fit to the exact function P, whose value depends on the direction of waves (encoded in the k_x, k_y variables), the properties of the medium ($v_p, \varepsilon, \delta, \ldots$) as well as the value of the reference velocity v_0. Typically the resulting approximation is accurate at small and medium angles, but accuracy is degraded as the angle between the direction of propagation and the vertical increases. For illustration, we show in Figure 1 the value of the function P as well as the Padé approximation to it, for an angle $\theta = 50°$ as a function of the azimuth angle of the propagation direction. The medium is a TTI one with $\varepsilon = 0.2, \delta = 0.05$; the axis of symmetry is specified by polar and azimuth angles $\theta_a = 30°, \varphi_a = 0$. We note the errors are generally larger in the direction of the TI axis of symmetry ($\varphi = 0$) as well as in directions in-between the 4 axes used for spitting. Also, the values of the function, as well as the errors, increase with the ratio of the velocity to the reference velocity $r = v/v_0$.

PSPI step

The FFD method with optimized Padé coefficients provides usually a good solution for the one way wave equation in TTI media. However, there are some errors in the method. Some of these are due to approximations inherent in the 4-way splitting approach to estimating the quantity P (these depend on the azimuthal angle φ), others are due to the low order of the Padé polynomials (and will vary with the polar angle θ). Generally the errors are larger at large polar angles, but their magnitude also increases with the difference between the reference velocity v_0 and the actual medium velocity v. In areas where velocity varies laterally (for example, near sharp dips in the water bottom, or zones with gas clouds where the local velocity is low), the errors in a standard inplementation of the FFD method may become significant.

This gives the motivation to improve the accuracy in case of large velocity variations ($v_p \gg v_0$). We propose a method consisting of 1) propagating the wavefield using a smoothed background velocity and 2) using a FD operator to compensate for the difference between the true velocity and the background one, which is generally much smaller than $v_p - v_0$.

For the first step we propose to use the phase-shift plus interpolation method (PSPI). This is one of the early methods used for downward continuation; its strength lies in its simplicity and ease of implementation. We follow the procedure proposed by Gazdag, (1984). The method relies on interpolating between two phases computed at two constant velocities v_1 and v_2

$$\phi_{1,2} \;=\; \frac{\omega dz}{v_{1,2}}\left(\sqrt{1 - \left(\frac{v_{1,2}\,k_r}{\omega}\right)^2} \;-\; 1\right) \;, \quad (8)$$

with k_r the radial wave number, to obtain the phase at an intermediate velocity $v(x)$:

$$\phi_a(v) \;=\; a\phi_1 + (1-a)\phi_2 \;. \quad (9)$$

We note the vertical phase shift $\omega dz/v$ is subtracted from the phases ϕ so that for downward-propagating waves ($k_r = 0$) the phases are zero. Moreover, at small angles, the phases are proportional to the velocity $\phi \sim v$, therefore standard linear weighting $a = a_0 = (v_2 - v)/(v_2 - v_1)$ is a natural choice for the interpolation formula. However, it is possible to obtain a better fit over a larger angles range by a least square minimization of the difference between the exact phase at $\phi(v)$ and the interpolated phase $\phi_a(v)$ over some chosen range of k_r (we chose $\tilde{k}_r(v_2) = v_2 k_r/\omega < 0.95$).

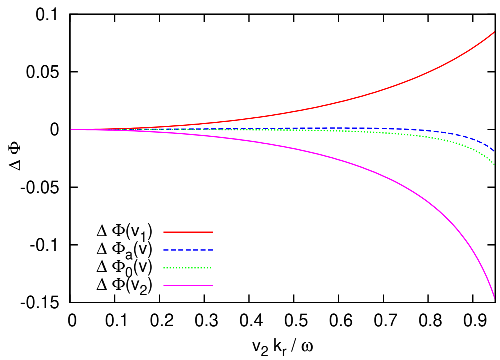


Figure 2: Error in interpolation of the phase $\phi = \phi_a(v) - \phi(v)$. The dashed line corresponds to least square value for a, while the dotted line corresponds to linear interpolation a_0. The top and bottom lines are differences for the reference velocities $\Delta\phi = \phi(v_i) - \phi(v)$.

Figure 2 shows the effect of the least square fit on the accuracy of the phase interpolation (9). The ratio for the velocity values picked for display are $v_2/v_1 = 1.5$, $v/v_1 = 1.25$. A weight value $a = 0.55$ is obtained from the least square fit; for comparison, the interpolated phase with the linear weighting $a_0 = 0.5$ is also shown. One can see that least square optimization significantly improves the match over a larger angles range. An estimation of the optimized weight a is given by the formula

$$a \;=\; a_0(1 + (1 - a_0)(-0.1 + 0.2v_2/v_1))\;. \qquad (10)$$

FD details

The FD part is applied following the PSPI step; formally

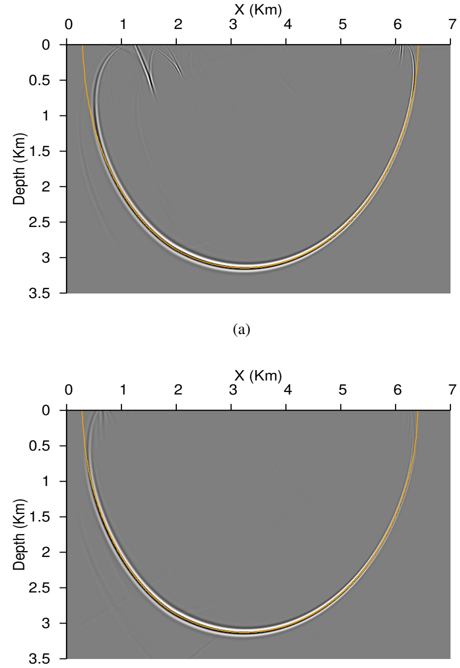
$$e^{ik_z dz} \;\simeq\; e^{ik'_z dz}\, e^{i\left[\omega\left(\frac{1}{v_z(x)} - \frac{1}{v_{sm}(z)}\right) + \Delta k'_z\right] dz}\,, \qquad (11)$$

where k'_z is the wavenumber for the PSPI velocity $v_{sm}(z)$. The first exponential on the right hand side is evaluated using PSPI, and the second using FD. A Padé expansion using four independent terms (Tang et al. 2019) is used for approximating the angle dependent $\Delta k'_z$ term. Since $v(x) - v_{sm}(x) < v(x) - v_0$, the magnitude of this term is less than that of original Δk_z in Eq. (3), and the errors in the computation of the FD term will be smaller. Note that, for the purpose of efficiency, in our proposed method in anisotropic media, PSPI is applied using an isotropic model only, the anisotropic impact for both VTI and TTI cases is taken into account in the FD step.

Another source of errors in the FD step is due to replacing the spatial derivative $\partial/\partial x_j$ with a finite difference operator. For simplicity and efficiency in solving the resulting banded diagonal system, one typically uses a second order difference operator

$$\begin{aligned} \frac{\partial}{\partial x}u \quad &\to \quad \frac{u_{j+1} - u_{j-1}}{2dx} \\ \frac{\partial^2}{\partial x^2}u \quad &\to \quad \frac{u_{j+1} - 2u_j + u_{j-1}}{dx^2}\,, \end{aligned} \qquad (12)$$

which has significant dispersion if the grid sampling dx is of order 2 points per wavelength. One solution to this problem is



(a)

(b)

Figure 3: Impulse response in a smoothly laterally varying velocity model (a) using the FFD method and (b) using PSPI + FD.

to replace the exact $\tilde{k}$ wavenumber variables in Eq. (5) by the "effective" wavenumber as measured by the applying the finite difference operator

$$\tilde{k}_j = \frac{k_j}{\omega s} \;\to\; \tilde{k}^e_j = \frac{2}{\omega s dx_j}\sin\left(\frac{k_j dx_j}{2}\right)\,. \qquad (13)$$

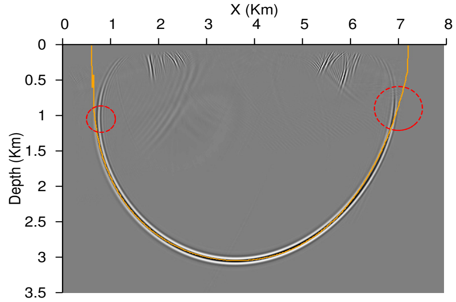
One could then recompute the coefficients $a_j, b_j, \ldots$ by performing a least squares match with these variables. However, as the results will depend on scale factors $h_j = \omega s dx_j$, this may not be feasible in practice; usually the Padé coefficients are precomputed and saved to/read from disk, and this approach would significantly increase the size of the coefficient tables. Instead, using the Taylor expansion of the Padé polynomial, we found the following corrections in leading order:

$$\begin{aligned} a_j \;&\to\; a_j + (c_j^2/a_j)h_j^2/24 \qquad & b_j \;&\to\; b_j + h_j^2/12 \\ c_j \;&\to\; c_j & d_j \;&\to\; d_j - (c_j/a_j)h_j^2/24\,. \end{aligned} \qquad (14)$$

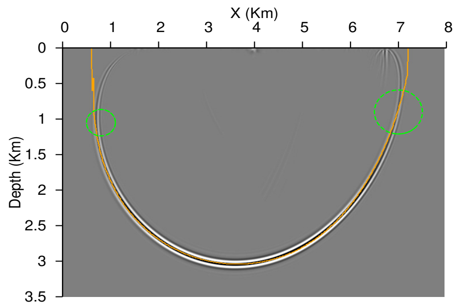
A stable and accurate implementation is obtained by applying the correction just to the the coefficients b_j; the corrections due to the linear terms (proportional to the coefficients c_j) are numerically small.

EXAMPLES

We show first an impulse response computed in a smooothly varying isotropic medium. The velocity changes linearly from $v_2 = 4$ Km/s at $x = 2$ Km to $v_1 = 2$ Km/s at $x = 6$ Km (the



(a)



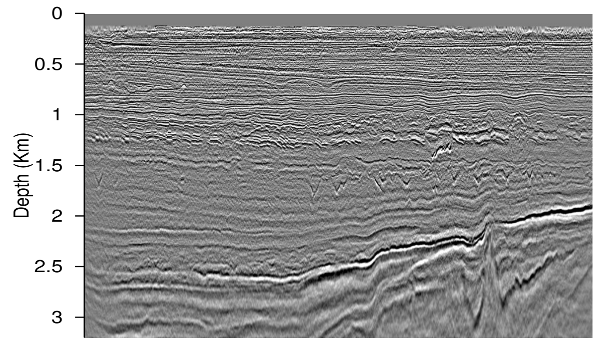
(b)

Figure 4: Impulse response in a 3d TTI model, along the axis of symmetry ; (a) using the FFD method and (b) using PSPI + FD method. The circles show areas where the accuracy has improved.

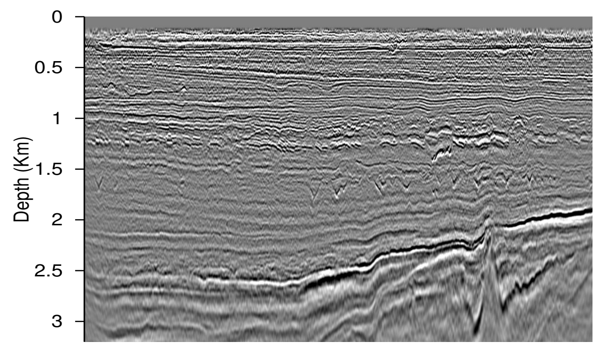
source location is at $x = 4$ Km). The top plot (Figure 3a) shows the impulse response computed with FFD; the overlay curve represents the maximum amplitude location computed with a 2 way propagator. One can note the errors in traveltime are larger on the left side of the image, where the velocity is larger. Adding a PSPI step, with a PSPI velocity at 95% of true velocity, leads to the result in Figure 3b. The large angle errors have been significantly reduced.

The next example shown in Figure 4 is for a homogenous TTI model, with $v_p = 4$ km/s and same anisotropic parameters used for Figure 1. We use a reference velocity of $v_0 = 2$ Km/s, introduced at a location outside image range. The top image (Figure 4a) is obtained using the FFD method only. The slice shown is along the TI axis of symmetry (x direction), where errors tend to be largest. By comparison with the RTM result (the overlayed orange line) we see there is good agreement except for angles of propagation close to the horizontal. Figure 4b shows the result with PSPI + FD. The background velocity used is $v_{sm} = 3.6$ km/s, and the two velocities used as reference for the PSPI interpolation are $v_1 = 3.3$ Km/s and $v_2 = 4$ km/s. The results shows an improvement in accuracy, as well as a reduction of noise, for large angle propagation.

We finally show a migration of an OBN line from the Utsira survey in the North Sea. The velocity model used for the area is based on long offset diving wave and reflection FWI (Jansen et al., 2021). The underlying model is TTI, with the tilt of the axis of symmetry reaching 60° near the high dip structures.



(a)



(b)

Figure 5: Migrated stack of OBN data a) WEM image b) RTM image

Figure 5a shows a 80 Hz WEM migration using the method proposed in this paper; the bottom image Figure 5b is the result generated with a 50 Hz RTM. The RTM result shows that the events on the WEM section are accurately positioned.

CONCLUSIONS

We present a method of improving the accuracy of one-way wave equation solutions in media with large lateral velocity variations. The standard FFD method, while highly accurate in most scenarios, still has some residual errors. These errors are magnified when the difference between the constant reference velocity (minimum velocity in an area) and the local velocity is large. We propose using PSPI with a smoothly varying model to replace the constant reference velocity based phase shift. As the PSPI velocity is closer in value to the local velocity, the errors inherent in the following FD corrections are reduced.

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